Contributed paper

OPTO-ELECTRONICS REVIEW **10**(1), 43–46 (2002)

Soliton like solutions and subsurface behaviour of the nematic layer

A. WALCZAK*

Institute of Applied Physics, Military University of Technology, 2 Kaliskiego Str., 00-908 Warsaw, Poland

The direct measurement of the refraction index profile in the nematic layer (NL) creates possibility to verify and exploit the non-linear solution of the Ericksen-Leslie (E-L) equation. It has been done for NL of 6CHBT tuned in wide range of external voltage. The symmetrical case of the director field distribution has been analysed. The way for local values of the electric field estimation inside the liquid crystal layer is discussed.

Keywords: nematic layer, soliton, internal electric field, 6CHBT.

1. Introduction

Earlier exploited method for refractive index profile measurement inside liquid crystalline (LC) waveguiding layer [1] is shown as a way for determination of the electric field distribution in this layer. Because refractive index measurement method has been already published, only most important details are repeated herein.

The exact analytical solution of the Ericksen-Leslie (E-L) equation in the single Frank elastic constant approximation is derived and, together with experimental results, applied for electric field determination. Similar solution was described in the literature for non-constrained space filled with LC [2,3]. Here, the soliton like solution for nematic layer (NL) of the thickness d has been derived. It has been shown that solution of E-L equation can be obtained in the closed form for the case of symmetrical deformation of the director field in the NL. As it is static solution it may be called "frozen" soliton like deformation induced in NL by an external electric field.

Finally, the obtained internal electric field values in the NL seems to deliver deeper view of the LC layer deformations. Those results may be useful information for charge movement investigations in the NL.

2. Refractive index measurement by means of the LC waveguide

In the prism coupler method applied in LC waveguide the guided modes propagation constant has been measured (see Fig.1) [4].

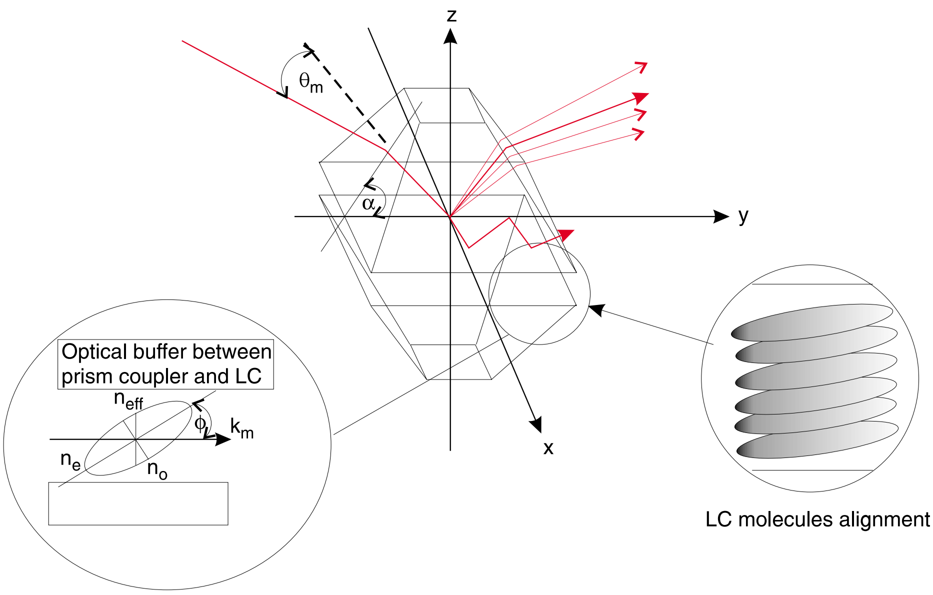

Fig. 1. Scheme of the idea of prism coupler method applied with LC waveguide. The optical buffer between prism and the LC layer in Fig. 1 consists of polyimide/SiO_2/ITO stacked layers.

*e-mail: awalc@ias.wat.waw.pl

$$N_m = n_{eff}\frac{\omega}{c} = n_p \sin\left[\alpha + \arcsin\left(\frac{\sin(\theta_m)}{n_p}\right)\right] \qquad (1)$$

In Eq. (1) α is the prism angle and θ_m is the incident angle for waveguide mode excitation. The wave number in free space is equal to ω/c. Both angles are easily measured. The m distinguishes modes in the waveguide. Mode in the waveguide shown in Fig. 1 is excited only in the case of equality between measured propagation constant and one of the possible mode's wavenumber [4]. Profile of the refraction index inside the guided layer my be preliminary estimated by

$$n(z) = N_m + \left(\frac{N_{m-1} - N_m}{z_m - z_{m-1}}\right)(z_m - z) \qquad (2)$$

In Eq. (2), z_m are the points of internal reflection of the guided mode m inside a waveguide. Those points, called turning points, are precisely determined by means of the following recurrence formula

$$\begin{aligned}\ln(z_m - z_{m-1}) = \\ & -\frac{1}{2}\ln\left[N_m\left(N_{m-1} - \sqrt{N_{m-1}^2 - N_m^2}\right)\right] + \frac{N_{m-1} - N_m}{N_m^2}\left(m + \frac{1}{2}\right)\pi - \\ & -\frac{N_{\mu-1} - N_m}{N_m^2}\sum_{k=1}^{k=m-1}\frac{1}{2(N_{k-1} - N_k)} \times \\ & \times\left\{(z_{k-1} - z_k)\left(N_k\sqrt{N_k^2 - N_m^2}\right) + N_m^2 \ln\left[\left(N_k - \sqrt{N_k^2 - N_m^2}\right)(z_k - z_{k-1})\right]\right\} + \\ & +(z_k - z_{k-1})\left(N_{k-1}\sqrt{N_{k-1}^2 - N_m^2}\right) + N_m^2 \ln\left[\left(N_{k-1} - \sqrt{N_{k-1}^2 - N_m^2}\right)(z_k - z_{k-1})\right]\Big\}\end{aligned} \qquad (3)$$

It is easy to see that measurement of N_m family for the observed waveguide modes is enough to obtain turning points position as well as $n(z)$ form finally. In fact (see Fig. 1), $n(z)$ is equivalent to n_{eff} which is seeing by each mode between its turning points. A known formula describes the effective refraction index

$$n_{eff} = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\phi(E,z) + n_e^2 \cos^2\phi(E,z)}} \qquad (4)$$

The value E in Eq. (4) is the local value of the driving field in the point z while the angle ϕ is shown in Fig. 1, and it is measured between the wave vector and optical axis direction. The waveguide application during measurement assures that direction of the wave vector is always the same and always along LC layer.

Values of the refractive index in discrete collection of guided modes turning points in the 3 µm thick NL waveguide of 6CHBT (4-trans-4-n-hexyl-cyclohexyl-isothiocyanatobenzene) are as in Fig. 2. Those values are applied further for determination local electric field in turning points.

3. "Frozen" soliton as solution of E-L equation

Ericksen-Leslie equation in single Frank constant approximation is like below, and is similar to sine-Gordon equation

$$\frac{d^2\phi}{dz^2} = -\frac{\varepsilon_0 \Delta\varepsilon E^2}{2K_F}\sin 2\varphi(z) \qquad (5)$$

It is one of the differential equations of the form

$$\frac{dy}{dz} = cf(y). \qquad (6)$$

The coefficient before *sine* in Eq. (5) is denoted by c. Simple integration gives

$$\frac{dy}{dz} = \pm\sqrt{2c\int f(y) + C_1}. \qquad (7)$$

When symmetric planar LC layer is deformed then condition

$$\left(\frac{dy}{dz}\right)^2 = 0, \qquad (8)$$

have to be fulfilling in the middle of the layer and C_1 can be put equal to 0. Finally the exact analytical solution of E-L equation in symmetric case is of the form of two integrals

$$\varphi(z) = \arctan\left[\exp\left(\sqrt{\frac{\varepsilon_0 \Delta\varepsilon E^2}{K_F}}(\pm z + C_2)\right)\right] \qquad (9)$$

In that form angle is measured from the normal direction in reference to LC layer. If hard anchoring $\phi(z = \pm d/2) = \pi/2$ is desired then $C_2 = d/2$. In Eq. (9), K_F is the Frank elastic constant, ε_0 is the dielectric constant, $\Delta\varepsilon$ is the dielectric anisotropy, E is the local electric field ampli-

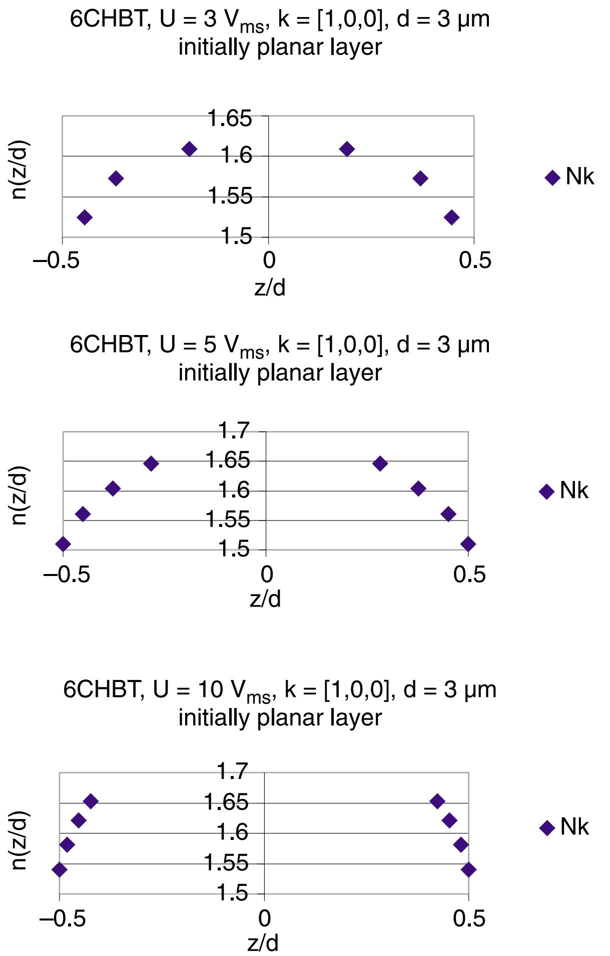

Fig. 2. Refractive index profile $n(z_k)$ in the turning points z_k obtained in 6CHBT (4-trans-4-n-hexyl-cyclohexyl-iso-thiocyanatobenzene) tuned waveguide for different external rms electric field (after Ref. 5).

tude. It is soliton like solution of E-L equation. As time independent it describes "frozen" soliton. Full solution of E-L in the LC layer is the sum of both integrals (9). As we know, the electric field E in Eq. E-L, and also in the form of Eq. (9), is a local value of the field inside LC layer, at the point of differentiation. That value is unknown. It depends on external field value and on the dielectric properties of the NL. As anchoring of LC molecules on polyimide aligning layer influences dielectric properties of the thin NL then it is worthwhile to determine electric field inside that layer even in chosen, particular case.

4. Electric field inside LC layer – discussion of connections

From the experiment one can obtain n_{eff} values in the turning points of the guided modes in 3-μm thick layer of 6CHBT (see Fig. 2). So, in those points Eq. (4) with the angle ϕ, described by exact solution of E-L equation should mirror true electric field value across the LC layer. As that solution is stated for strong anchoring condition, so the results are good enough only in such a situation. For known 6CHBT properties [6] those values have been fitted in each turning point separately. In Fig. 3, the obtained local values of internal electric field in LC layer of 6CHBT are exhibited. Used 6CHBT substance has been prepared as high resistivity LC in Prof. Dąbrowski's laboratory in Military University of Technology. Its resistivity has been estimated as higher then 10^9 Ω m.

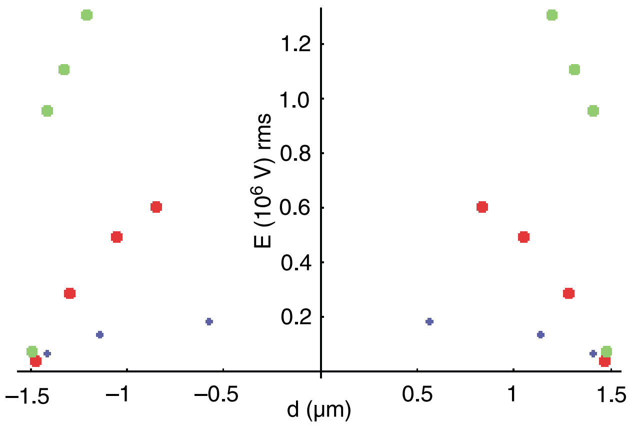

Fig. 3. Electric field values in turning points for different driving voltages: light grey 10 V, black-5 V and grey small dots-3 V in 3 μm LC layer of 6CHBT.

The shape of the obtained electric field distribution is similar, to some extent, to induced rotation of local optic axis mirrored in refractive index variation across the layer. Such result has been expected. It directly illustrates the existence of the electric field gradient in LC layer edge vicinity. Unfortunately, in a prism coupler method, the guided mode turning point position nearest to LC layer boundary is usually loaded with the highest error. So, extrapolation of the electric field towards the edge of the layer contains that error as well. It is the reason for presentation of the electric field values only in discrete points without extrapolation.

5. Conclusions

The presented results are a part of permanent discussion about macroscopic description of electric fields inside thin LC layer. It is important for wide range of LC applications. The aim of that work was extracting those coincidences between important macroscopic properties of the NL, which should be taken into account during determination of the electric field distribution inside LC layer. Determination of that field seems to be important especially for investigation charge movement and distribution inside the LC layer. It has been done for 6CHBT thin layer with strong anchoring condition and possible highest resistivity.

The analytical solution of E-L equation, as well as direct measurement of the refraction index profile in flat cross-section of LC layer, allowed us to do this. According to the author knowledge, the presented exact solution of the E-L equation in the NL with the constrained thickness d

has not been derived earlier. That solution seems to be useful for detailed study LC permittivity as well as near boundary LC behaviour.

It turned out that boundary value of tilts (so real anchoring conditions) and LC permittivity as well as Frank constant of LC is permanently connected with a shape of internal electric field [see formula Eq. (9)]. Extrapolation of the electric field distribution towards the layer edge must be built on deeper experimental analysis, because of lack of guaranteed refractive index data immediately at the edge of the layer.

Acknowledgements

This work was supported by the grant of MUT no. PBW751.

References

1. A. Walczak, “New method for analysis of liquid crystal elasticity”, *Acta Phys. Pol.* **A94**, 5–6 (1998).
2. S. Lam and A. Prost, *Solitons in Liquid Crystals,* Springer-Verlag, New York, 1991.
3. N. Andal and G.S. Ranganath, “Structure of planar solitons in nematic and smectic liquid crystals”, *Phys. Rev. Lett.* **20**, 321–330 (1996).
4. R.T. Kersten, “The prism coupler as a precision instrument”, *Optica Acta* **22**, 503–513 (1975).
5. A. Walczak, *“Investigation of elasticity of nematics using liquid crystal waveguides”*, Warsaw, MUT, 1999. (in Polish).
6. J. Kędzierski, *“Elasticity, diamagnetic susceptibility and diamagnetic potential and potential of nematic interaction with a substrate”*, Warsaw, MUT, 1994. (in Polish).

