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# Evolution of light bullets propagating in saturable Kerr-like media with higher order effect

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*In the paper, propagation of spatio-temporal pulses (light bullets) in inhomogeneous, Kerr-like nonlinear medium is considered. For ultra-short bullets the higher-order nonlinear effects – self frequency shift and nonlinear dispersion together with saturation of nonlinear susceptibility are taken into account. The equations describing evolution of temporal and spatial width of the bullet parameters are derived. The stationary solutions of these equations are found analytically. Small oscillations around the stationary bullet's widths are considered and frequency oscillations are obtained. The regime of stable oscillations is discussed.*

**Keywords:** light bullets, spatio-temporal pulses, temporal solitons, higher-order nonlinear effects, saturable Kerr media.

## 1. Introduction

Localized in time and space optical pulses – light bullets are the objects of increasing interest in telecommunication systems. Such bullets transmitted for long distance can be treated as bits of information. But two fundamental optical effects diffraction and dispersion broaden the pulses what can cause losses of information. In nonlinear Kerr medium, however, the additional effect of self phase modulation (SPM) can diminish or even completely suppress dispersion or diffraction [1–3]. In pure Kerr medium, such a balance between SPM and dispersion or SPM and diffraction occur in $1+1$ dimensions. Nevertheless, the suppression of both diffraction and dispersion in $3+1$ dimensions is possible. Such a total compensation of two effects causing broadening of the pulse can be achieved in Kerr-type saturable media when propagating fields are sufficiently strong [4–6].

In linear telecommunication lines, spatial broadening of the pulse in $3+1$ dimensions is suppressed otherwise, by means of appropriate distribution of linear refractive index. Additionally, if temporal broadening caused by dispersion is compensated by nonlinearity of the medium, we obtain light bullets that can propagate in the nonlinear Kerr medium without saturation [1] or when saturation of the medium is quite weak. This case we meet in practice even for ultra-short pulses. However, describing propagation of such pulses we should take into account a few additional higher order effects. Let us neglect the linear effect of the third order dispersion, it complicates calculations very much. Nevertheless, we will consider two other effects, both nonlinear, nonlinear dispersion (ND) and self frequency shift (SFS, nonlinear Raman effect) [1,7–10].

Unfortunately, the higher-order nonlinear Schrödinger equation (HONSE) describing propagation of the pulse envelope [8–10] has no analytic solutions if we assume $(3+1)$-dimensional case, linearly non-uniform medium, saturation and higher-order nonlinear effect. Even numeric solution of such an equation in $(3+1)$-dimensions is difficult and time-consuming. Nevertheless, using a sort of variational method [4,10–15] we can obtain an approximate solution of this equation. This method applies Lagrange equations formalism and is very convenient because we can assume a given input field and follow the changes of field's parameters during propagation. This method of investigation was applied to light bullets in graded-index Kerr medium [15], but the case considered in our case has been never analysed.

## 2. Propagation of light bullets in generalized nonlinear media

Let us consider an optical pulse travelling along the $z$-axis in the infinite nonlinear Kerr-like medium. The pulse envelope is described by the function $U(x,y,z,t)$ slowly varying in time and along the propagation direction. In the medium with linear refractive index $n(\omega)$, the numeric value of group velocity dispersion coefficient $k_2 = \mathrm{d}^2[\omega n(\omega)/c]/\mathrm{d}\omega^2$, taken for the carrying frequency $\omega$, determines whether temporal width of the pulse increases during propagation ($k_2 \neq 0$) or not ($k_2 = 0$). Another effect – dispersion increases the transverse widths of the pulse in $x$ and $y$ directions but this effect acts ever in linear, uniform medium.

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Let the pulse propagates through a nonlinear Kerr-type medium with nonlinear part of permittivity depending on the field intensity $\varepsilon_{NL} = \varepsilon_{NL}(|U(x,y,z,t)|^2)$. In this case, the pulse envelope satisfies $(3+1)$D nonlinear Schrödinger equation [1,4,6,15]. To suppress dispersion, suppose that the linear part of the medium permittivity $\varepsilon_L = n^2(\omega)$ creates a sort of trap for the pulse. Such a trap can be formed by a proper transverse profile of the function $\varepsilon_L = \varepsilon_L(x,y)$. In order to perform analytic calculation let us assume the square graded-index profile. Moreover, for the convenience let us introduce the factor $2/k$ multiplying the nonlinear permittivity function $\varepsilon_{NL}$. As a result, the total permittivity will be expressed by the function

$$\varepsilon(x,y,z,t;\omega) = n_0^2(\omega) \times\left(1-\Omega^2(x^2+y^2)+\frac{2\varepsilon_{NL}(|U(x,u,z,t)|^2)}{k}\right). \tag{1}$$

The above form generalizes permittivity considered by Raghavan *et al.* [15] into the case of non-Kerr, but Kerr-like media. The nonlinear permittivity describing saturation of nonlinearity in cubic-quintic model [4,6] gives $\varepsilon_{NL}$ in the form

$$\varepsilon_{NL} = \varepsilon_2\left(|U|^2 - \frac{|U|^4}{I_s}\right). \tag{2}$$

The parameter $I_s$ estimates the strength of the field for which saturation should be taken into account. In the pure Kerr medium $I_s = \infty$, but even for the considered strong fields we shall assume $|U^2| << I_s$. The equation describing propagation of the pulse envelope $U(x,y,z,t)$ in a frame moving with the group velocity $1/k_1 = 1/\{d[\omega n(\omega)/c]/d\omega\}$ is called $(3+1)$D higher-order (generalized) nonlinear Schrödinger equation (HONSE)

$$\frac{\partial U}{\partial z} + \frac{ik_2}{2}\frac{\partial^2 U}{\partial t^2} - \frac{i}{2k}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)U + \frac{ik\Omega^2(x^2+y^2)U}{2} - i\varepsilon_{NL}(|U|^2)U + \gamma\left(2\frac{\partial}{\partial t}\varepsilon_{NL}(|U|^2)U + (\kappa-1)U\frac{\partial}{\partial t}\varepsilon_{NL}(|U|^2)U\right) = 0. \tag{3}$$

Two nonlinear higher-order effects, i.e., nonlinear dispersion and self frequency shift are included in two last terms of this equation. The parameter $\gamma$ before these terms specify the magnitude of both higher-order effects while relation between them: $\kappa = 1$ for ND only and $\gamma\kappa \to const$ for $\gamma \to 0$ if ND vanishes. The form of Eq. (3) generalizes and combines equations considering in many papers [4,6,10–17].

To find an approximate solution of Eq. (3) describing light bullet let us write the Lagrange density function equivalent to HONSE Eq. (3). After calculation we can convince that this function has the form

$$\begin{aligned}\mathscr{L} = {} & \frac{i}{2}\left(U\frac{\partial U^*}{\partial z} - U^*\frac{\partial U}{\partial z}\right) + \frac{1}{2k}\left(\left|\frac{\partial U}{\partial x}\right|^2 + \left|\frac{\partial U}{\partial y}\right|^2\right) \\ & - \frac{k\Omega^2(x^2+y^2)|U|^2}{2} - \frac{k_2}{2}\left|\frac{\partial U}{\partial t}\right|^2 - F(|U|^2), \\ & - \frac{i\gamma}{2}\left(\frac{(\kappa-1)}{|U|^2}F(|U|^2) - (\kappa+1)\varepsilon_{NL}(|U|^2)\right)\end{aligned} \tag{4}$$

where $F(|U|^2)$ is an integral of nonlinear permittivity function [4,16–17]

$$F(|U|^2) = \int_0^{|U|^2}\varepsilon_{NL}(I)dI = \varepsilon_2\left(\frac{|U|^4}{2} - \frac{|U|^6}{3I_s}\right). \tag{5}$$

The last part of this formula contains a form of the function $F(|U|^2)$ specific for cubic-quintic nonlinearity, Eq. (2).

We assume the trial function $U(x,y,z,t)$ describing envelope of the bullet in a form of the product of two terms, one depending on the transverse coordinates $(x,y)$ but not depending on the time $t$ and the other, depending on $t$ but not on $x$ and $y$, $U(x,y,z,t) = R(x,y,z,t)u(z,t)$. The first function in this product $R(x,y,z)$ is the solution of the transverse part of Eq. (3) that is the solution of equation describing propagation of the field in linear self-focusing medium. Solving this equation we obtain gaussian beam of the constant width $a(z) = 1/\Omega$, $R(x,y,z) \sim \exp[-\Omega^2(x^2+y^2)/2]$ [18]. Similarly, we can also write a solution of the longitudinal part of Eq. (3), i.e., $(1+1)$D HONSE. This equation was considered in Refs. 16 and 17 and has the solution for any dielectric function $\varepsilon_{NL}(|U|^2)$. Nevertheless, the general solution of this equation can be written only in quadrature, for the considered case $I_s \to \infty$ and small higher terms of this quadrature gives the explicit formula. The obtained profile of the bright soliton is similar to that in Kerr medium: $|u(z,t)|^2 \sim 1/[\cosh^2(t/T)+\sigma]$. The broadening parameter $\sigma$ appearing here is small, it depends on the small $\gamma^2$ and $1/I_s$

$$\sigma = \frac{1}{T^2}\left(\frac{\gamma^2(\kappa^2-4)}{3} - \frac{2k_2}{3\varepsilon_2 I_s}\right). \tag{6}$$

This parameter describes deviation of the shape of soliton in Kerr-like saturable medium with ND and SFS from the Sech profile. For $\sigma > 0$, the resulting solitons are higher and shorter than solitons propagating in pure Kerr medium, for $\sigma < 0$ this relation is inverse.

 

The form of transverse and longitudinal solutions suggests taking the trial solution of Eq. (3) describing (3 + 1)D light bullet as

$$U(x,y,z,t) = \sqrt{\frac{3}{2\pi Ta^2(3-2\sigma)}\frac{P_0}{\cosh^2(t/T)+\sigma}} \times\exp\left(-\frac{x^2+y^2}{2a^2}+i\Phi(x,y,z,t)\right). \tag{7}$$

In the above formula, $P_0$ is the total power carried by the bullet

$$P_0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|U(x,y,z,t)|^2\,dxdydt, \tag{8}$$

and the broadening parameter $\sigma$ is given by exactly the same Eq. (6) as in the soliton case. Nevertheless Eq. (7) is constructed from exact solutions of two parts of Eq. (3), it does not satisfy its full form, Eq. (3). Therefore both the temporal and spatial widths $T$ and $a$ together with the broadening parameter $\sigma$ should change during propagation $T = T(z)$, $a = a(z)$, and $\sigma = \sigma(z)$. Consequently, similar terms changing with $z$ will appear in the phase $\Phi(x,y,z,t)$ of the bullet. We shall assume three such terms, two of them describe gaussian curvature and chirp while the third one all other effects influencing the phase but independent on $x$, $y$, and $t$

$$\Phi(x,y,z,t) = -\frac{\gamma(\kappa+2)}{T}\tanh\left(\frac{t}{T}\right), + (x^2+y^2)\alpha + t^2\theta + \varphi \tag{9}$$

with $\alpha = \alpha(z)$, $\theta = \theta(z)$, and $\varphi = \varphi(z)$. The first term in Eq. (9) approximates the phase of bright soliton in Kerr-like medium with ND and SFS [17].

The Lagrange density function $L$ with $U(x,y,z,t)$ given by Eq. (7) can be integrated over $x$, $y$, and $t$. Doing so we obtain the Lagrange function $L$ depending only on $z$

$$\begin{aligned} L = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathscr{L}(x,y,z,t)dxdydt &= P_0\Bigg(\phi' + a^2\left(\alpha' + \frac{2\alpha^2}{k}\right) \\ &+ T^2(\theta' - 2k_2\theta^2)\left(\frac{\pi^2}{12}+\frac{\sigma}{3}\right) + \frac{1+k^2\Omega^2a^4}{2ka^2} \\ &+ \frac{2\varepsilon_2P_0^2}{135\pi^2I_sa^4T^2} - \frac{4k_2}{3T^2}\left(\frac{1}{8}-\frac{\sigma}{15}+\frac{\gamma^2(\kappa+2)^2}{5T^2}\right) \\ &- \frac{\varepsilon_2P_0}{3\pi a^2T}\left(\frac{1}{4}-\frac{\sigma}{15}+\frac{\gamma^2(\kappa+2)(\kappa+3)}{5T^2}\right)\Bigg), \end{aligned} \tag{10}$$

where prime $'$ denotes differentiation with respect to $z$. Treating $\phi(z)$, $a(z)$, $T(z)$, $\alpha(z)$, and $\theta(z)$ as a set of generalized coordinates of the bullet we can write the Euler-Lagrange equations describing their evolution, i.e., the propagation. In this way, we shall derive five coupled first-order differential equations. It can be proved that the function $\phi(z)$ does not appear in any of the obtained relations, the corresponding Euler-Lagrange equation simply states 0 = 0. Moreover, the phase terms $\alpha(z)$ and $\theta(z)$ and their derivatives can be eliminated from the other four relations giving two second-order differential equations for temporal and spatial widths $T(z)$ and $a(z)$

$$\begin{aligned} &\frac{\pi^2-4\sigma}{4k_2^2}T^2T'' - \frac{\sigma}{k_2^2}TT'^2 - \frac{1}{T} - \frac{\varepsilon_2P_0}{4\pi k_2a^2} \\ &+ \frac{4\varepsilon_2P_0^2}{45\pi^2k_2I_sa^4T} + \frac{\varepsilon_2P_0}{5\pi k_2a^2}\left(\sigma - \frac{3\gamma^2(\kappa+2)(\kappa+3)}{T^2}\right) \\ &+ \frac{16}{15T}\left(\sigma - \frac{3\gamma^2(\kappa+2)^2}{T^2}\right) = 0 \\ &a'' + \Omega^2a - \frac{1}{k^2a^3} + \frac{\varepsilon_2P_0}{6\pi ka^2T} - \frac{8P_0^2}{135\pi^2ka^5T^2I_s} \\ &- \frac{16\varepsilon_2k_2P_0^2}{45\pi ka^3T}\left(\sigma - \frac{3\gamma^2(\kappa+2)(\kappa+3)}{T^2}\right) = 0 \end{aligned} \tag{11}$$

Derived system of equations, Eq. (11), with the temporal broadening parameter $\sigma$, expressed by Eq. (6), constitutes the basis of further analysis of bullet's propagation in saturable medium with ND and SFS. The equations are fairly complicated, but their complexity is caused mainly by corrections that appeared because of saturation and higher-order effect. But the main profit achieved using Eq. (11) is the possibility of tracing the changes of these parameters of the bullet that have simple physical interpretation, what contrasts with description given by nonlinear Schrödinger equation. However, solving HONSE Eq. (3) we obtain the exact shape of an envelope, even having such a shape we cannot determine how the bullet widths evolve. Therefore description based on Lagrange Eq. (11) give us supplementary insight into evolution of the bullet.

## 3. Stationary propagation and small oscillations

Generally, the temporal and spatial width of light bullet change as the bullet moves through the medium. Nevertheless it is possible to fit these quantities in such a way, that they will remain constant during propagation. These stationary values $T_0$ and $a_0$ will be a solution of two algebraic equations obtained from Eq. (11) for vanishing the derivatives $T'(z)$, $T''(z)$ and $a''(z)$. The first equation obtained in this way will contain only two significant terms [the third and the fourth one in the first line of Eq. (11)], all others will be small corrections appearing because of saturation and the higher order effects. That is why we can solve this equation obtaining the stationary temporal width $T_0$ as the explicit function of the spatial width $a_0$

$$T_0 = -\frac{4\pi k_2 a_0^2}{\varepsilon_2 P_0} + \frac{\varepsilon_2 P_0}{45\pi a_0^2}\left(\frac{14}{\varepsilon_2 I_s} - \frac{\gamma^2(\kappa+2)(8\kappa-7)}{k_2}\right). \quad (12)$$

This value introduced into the second equation of Eq. (11) for the stationary state $T_z(z) = T_0$ and $a(z) = a_0$ gives the relation satisfied by the spatial stationary width $a_0$

$$\begin{aligned} & a_0^2 + \frac{k\varepsilon_2^2 P_0^2}{24\pi^2 k_2} - k^2\Omega^2 a_0^6 \\ & + \frac{k\varepsilon_2^3 P_0^4}{15\pi^4 k_2^3 a_0^4}\left(\frac{k_2}{9I_s} - \frac{\gamma^2(\kappa+2)\varepsilon_2}{k_2}\right) = 0 \end{aligned}. \quad (13)$$

The last terms in both Eqs. (12) and (13) express small corrections caused by saturation and higher order terms in HONSE. Without these corrections both Eqs. (12) and (13) have the same form as equations given in Ref. 15 describing stationary widths of light bullets in pure Kerr medium.

As equation determining $a_0^2$, Eq. (13) is the perturbed cubic algebraic equation. Nevertheless its solution can be written explicitly, its form is so complicated that we will not write it. Instead, we draw $a_0$ together with $T_0$ given by Eq. (12) versus the power $P_0$ carrying by the bullet (Fig. 1). The four lines for each of the curves $a_0(P_0)$ and $T_0(P_0)$ have been obtained for different values of the material coefficients $I_s$ and $\gamma$ ($\kappa = 4$ in all these cases). One of the lines is drawn for the pure Kerr medium ($I_s = \infty$, $\gamma = 0$), two other for the case of saturable medium without the higher order terms and generalized medium without saturation and the fourth one exhibit simultaneous influence of all considered effects. As we can see, increase in power carrying by the bullet gives significant decrease in its temporal width, this property is very important in view of the potential. Note that for the bullets with short temporal width in the regime of high power, influence of the higher order terms can be very significant, even very small values of the coefficient $\gamma$ give quite large decrease in the bullet's temporal width. Unfortunately, the sign of this change depends on relation between two higher order terms in HONSE (controlling by $\kappa$) and power of the bullet $P_0$. Moreover, the large value of $\Delta T$ compared with $T_0$ means that we are outside the regime of applicability of used approximations. Nevertheless numerical solutions of exact equations confirm that for many cases we can obtain significant decrease in bullet temporal width due to the higher order terms in HONSE.

Note that positive values of stationary temporal width can be obtained only when $k_2/\varepsilon_2 < 0$, it is the same condition that allows for solitons existence in pure Kerr medium. But the signs of $k_2$ and $\varepsilon_2$ have fundamental significance when we try to solve Eq. (13) and analyse behaviour of its solution in the limit $\Omega \to 0$. In the case of the focusing medium $\varepsilon_2 > 0$ and the regime $k_2 < 0$, Eq. (14) has finite solution $a_0$ (for finite power of the bullet $P_0$) even in the linearly uniform medium $\Omega = 0$. Unfortunately, these solutions are unstable with respect to small perturbation. Therefore we will assume the case of defocusing medium $\varepsilon_2 < 0$ with the positive group velocity dispersion $k_2 > 0$ (Fig. 1).

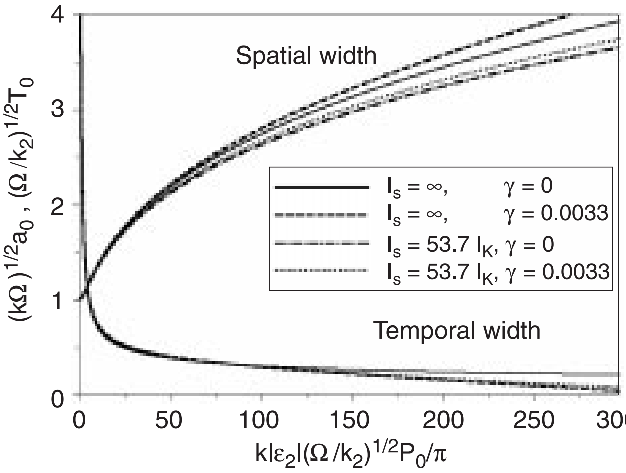


Fig. 1. Stationary propagation in media with $\varepsilon_2 < 0$, $k_2 > 0$, $\kappa = 4$. The temporal and spatial widths $T_0$ and $a_0$ as functions of the power $P_0$ carrying by the bullet.

This choice of signs causes that light bullet tends to soliton of constant temporal width in the limit $\Omega \to 0$. The spatial width of the bullet approaches infinity but the product $\Omega a_0$ remains constant. Power of the bullet $P_0$ also increases infinitely $P_0 \sim 1/\Omega^2$. This behaviour of solutions can be identified in Fig. 1. The limit $\Omega \to 0$ results when horizontal coordinate increases infinitely.

Having stationary solutions of Eq. (11) we can also describe propagation of light bullets if its temporal and spatial width slightly deviates from the stationary value

$$\begin{aligned} T(z) &= T_0 + \delta T(z) \\ a(z) &= a_0 + \delta a(z) \end{aligned}. \quad (14)$$

The above widths introduced into Eq. (11) give two equations describing evolution of the deviation functions $\delta T(z)$ and $\delta a(z)$. Assuming these deviations to be small we can neglect all the terms containing the second and the higher powers of $\delta T(z)$, $\delta a(z)$ and their products in the obtained expressions. As a result of this procedure we get the linearised equations of the form

$$\begin{aligned} \delta T''(z) &= M_{TT}\,\delta T(z) + M_{Ta}\,\delta a(z) \\ \delta a''(z) &= M_{aT}\,\delta T(z) + M_{aa}\,\delta a(z) \end{aligned}, \quad (15)$$

with $M_{TT}$, $M_{Ta}$, $M_{aT}$, and $M_{aa}$ being the functions of the stationary values $T_0$ and $a_0$

$$\begin{aligned} M_{TT} &= -\frac{4k_2^2}{\pi^2 T_0^4} + \frac{m_{TT}}{I_s} + l_{TT}\gamma^2 \\ M_{Ta} &= \frac{8k_2^2}{\pi^2 a_0 T_0^3} + \frac{m_{Ta}}{I_s} + l_{Ta}\gamma^2 \\ M_{aT} &= -\frac{4k_2}{3ka_0 T_0^3} + \frac{m_{aT}}{I_s} + l_{aT}\gamma^2 \\ M_{aa} &= -4\Omega^2 + \frac{m_{aa}}{I_s} \end{aligned}, \quad (16)$$

 

where $m_{TT}$, $m_{Ta}$, $m_{aT}$, $m_{aa}$, $l_{TT}$, $l_{Ta}$, and $l_{aT}$ are the certain quite complicated coefficients describing the first-order corrections.

Two equations of Eq. (15) describe small variations of bullets parameters. But a form of these equations enables us to interpret the changes of widths appearing during propagation as vibrations in a system of two coupled oscillators. In such a system, both oscillators vibrate with the same frequency and, in general, their vibrations are superpositions of two normal oscillations. Assuming a solution of equations system (15) in the form

$$\delta T(z) = e^{ifz}\delta T(0), \quad \delta a(z) = e^{ifz}\delta a(0) \tag{17}$$

with $\delta T(0)$ and $\delta a(0)$ defined by the initial conditions, we can find frequencies of normal oscillations

$$f^2 = -\frac{M_{TT}+M_{aa}}{2} + \sqrt{\left(\frac{M_{TT}-M_{aa}}{2}\right)^2 + M_{Ta}M_{aT}}, \quad f^2 = -\frac{M_{TT}+M_{aa}}{2} - \sqrt{\left(\frac{M_{TT}-M_{aa}}{2}\right)^2 + M_{Ta}M_{aT}}. \tag{18}$$

Introducing here $M_{ij}(i,j = T,a)$, expressed by Eq. (16), we can check that $-(M_{TT} + M_{aa})/2 > 0$, for $\varepsilon_2 < 0$ and $k_2 > 0$ the expression inside the square root symbol can be positive or negative, but nevertheless $(M_{TT} + M_{aa})^2 \geq (M_{TT} - M_{aa}) + 4M_{Ta}M_{aT}$. Therefore in a system described by Eqs. (15), two or none normal oscillations appear, however, this conclusion is valid only for $\varepsilon_2 < 0$ and $k_2 > 0$ and small changes caused by saturation and higher-order effects.

Figure 2 illustrates the relation between the frequencies of Eq. (18) and the power carrying by the bullet $P_0$ [Eqs. (12) and (13) are used to express $a_0$ and $T_0$ as the functions of $P_0$]. We can see the two ranges of bullet's power for which oscillations can exist separated by wide gap between them. Oscillating bullets with the power corresponding to lower regime resemble gaussian beam – they possess large rather temporal width (see Fig. 1) and quite narrow waist. Therefore much more interesting is the upper regime of possible solutions corresponding to the parameters characterizing short bullets. One can observe, that one of the frequencies in this regime increases significantly with increasing power of the bullet.

In Fig. 3, we can see oscillations of the bullet resulting when we solve HONSE Eq. (3) numerically without any additional approximations for the parameters almost corresponding to that used for drawing the dotted lines in Figs. 1 and 2. The obtained changes of both widths are periodic and during the normalized distance $\Omega z = 100$ both widths perform the same number of full periods of changes. The bullet power is $P_0 = 50\pi/(k|\varepsilon_2|)\sqrt{k_2/\Omega}$ in the normalized units, what corresponds to the value not far from vertex of this line, however, one of the resulting frequencies is more

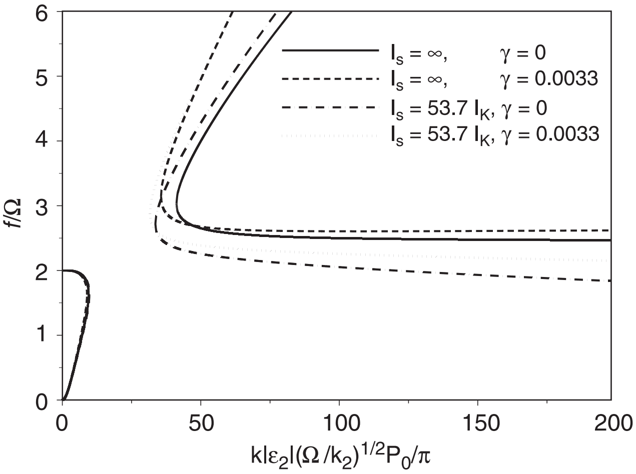


Fig. 2. Small oscillations around the stationary state. Periods of the normal oscillations $f$ as a function of the power $P_0$ carrying by the bullet.

than two times greater than the other. Note that the mean value of each width well corresponds to the values obtained from Fig. 1 for an appropriate power.

For the parameters of the bullet, corresponding to the values lying inside this gap in Fig. 2, the oscillations will be unstable. This unstable behaviour we can observe in Fig. 4. The bullet performs a few quite irregular oscillations and after them a temporal width increases infinitely. As a result, the bullet transforms into a gaussian beam propagating along the $z$-axis with regular, sinusoidal oscillations. The period of these oscillations obtained using numerical data equals $\pi$ (in normalized units) what exactly gives the value characteristic for oscillation of gaussian beam in self-focusing medium.

## 4. Conclusions

Small oscillations of light bullet propagating in non-uniform cubic-quintic saturable medium with higher order nonlinear effects can be described analytically. The de-

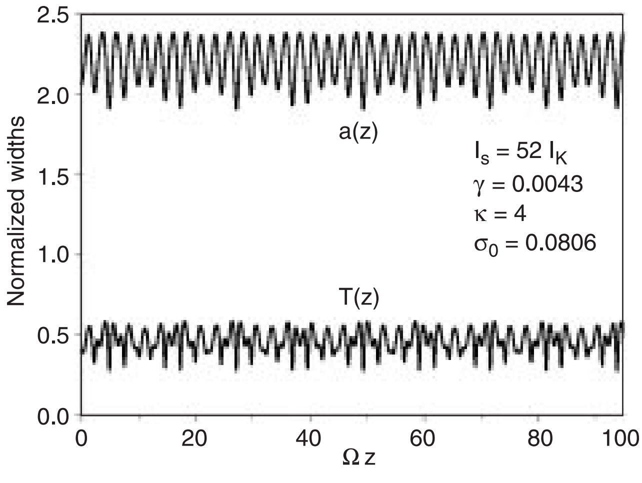


Fig. 3. Numeric solutions of the evolution equations. Case of stable propagation.

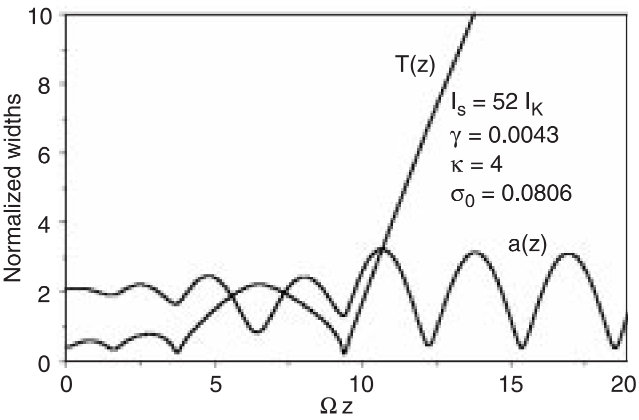


Fig. 4. Numeric solution of the evolution equations. Case of unstable propagation.

scription is based on Lagrange formalism and demands existence of stationary states, for which the bullet propagates without any changes. Both spatial and temporal widths of the bullet oscillate in the same manner. The amplitude of the field also oscillates and its value is adjusted so, that the power carrying by the bullet remain constant. Even if a magnitude of oscillations is not very small, the description is quite good.

The higher-order effects change both temporal and spatial width of the stationary bullet and for large power of the bullet even for small values of the higher order terms the resulting change can be quite large. In this way we can obtain a significant decrease in the bullet temporal width.

The relation frequency of oscillation versus bullet's power gives two possible values of frequency corresponding to any value of the power, but certain regime of the powers is forbidden. The forbidden regime covers all values of the power for media with the positive nonlinearity $\varepsilon_2$ and the negative group velocity dispersion $k_2$, while for the media with negative $\varepsilon_2$ and positive $k_2$ we meet a finite gap. For the powers from the forbidden regime, the oscillations of bullet are unstable. The gap became narrower in saturable media and media exhibiting the higher order nonlinear effects. These effects change also the frequencies of oscillations. The frequencies corresponding to the lower band change only slightly, but the frequencies in the higher band change quite well even for small higher order effects.

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