OPTO-ELECTRONICS REVIEW **13**(2), 135–140

Spatial solitons in biased photorefractive media with quadratic electro-optic effect

A. ZIÓŁKOWSKI* and E. WEINERT-RĄCZKA

Department of Electrical Engineering, Szczecin University of Technology,
17 Piastów Ave., 70–310 Szczecin, Poland

Propagation of optical beams and properties of one-dimensional (1D) spatial solitons in biased photorefractive media with quadratic electro-optic effect are analysed. An exact analytic solution of the corresponding propagation problem is presented as well as a numerical investigation of the evolution of optical beams.

Keywords: spatial solitons, beam evolution, photorefractive effect.

1. Introduction

Intensive studies of physical mechanisms leading to self-trapping of light have given rise to a great deal of nonlinear media which can support spatial solitons. Among many branches of these media, materials with saturable form of nonlinearity have found a growing interest in the past few years. Most of works in this domain are dedicated to the crystals with quadratic nonlinearity [1], saturable atomic vapours [2] and photorefractive materials [3,4]. However, in spite of considerable theoretical research, only in sparse cases self-trapping of light can be described by exact solvable models. Interesting form of saturation, leading to exact analytic solution of the corresponding propagation equation, has been proposed by Królikowski and Luther-Davies [5]. It turns out, that a modified form of this kind of nonlinearity can be used to analytical description of spatial solitons in photorefractive materials with quadratic electro-optic effect.

The refractive index change, produced by the quadratic electro-optic response to a photoinduced electric field, occurs, e.g., in centrosymmetric photorefractive materials [6]. This feature contrasts centrosymmetric media with their conventional noncentrosymmetric counterparts and makes them attractive, because of large attainable change in a refractive index. Up to now, photorefractive spatial solitons have been observed only in one material with this kind of nonlinearity, potassium lithium tantalite niobate (KLTN) [7], which was specially treated to have a first-order ferroelectric-paraelectric phase transition slightly below a room temperature. However, besides KLTN, photorefractive multiple quantum well (PRMQW) slab waveguide has been proposed to support spatial solitons at near-resonant wavelength [8]. In this case quadratic response to the electric field compensates tiny electro-optic coefficient, which is a general drawback of semiconductors, and permits self-trapping of light in materials which are much faster than other photorefractives.

Here, we analyse properties of one-dimensional (1D) spatial solitons within photorefractive media with quadratic electro-optic response. Because in Ref. 6 only numerical approach has been used, we develop analytical solvable model similar to the one proposed in Ref. 5 to describe propagation of light in such systems. Furthermore, we investigate numerically evolution of 1D Gaussian beams in these media and notice recurrent changing of their shape, similar to reported for conventional, nonitegrable form of photorefractive nonlinearity [9,10]. Simple numerical investigation of losses influence on soliton propagation is also made.

2. Theoretical model

We consider the externally biased photorefractive medium with the electric field applied along the x-axis. The electric field of 1D monochromatic optical beam, propagating along the z-axis can be expressed as

$$E_{opt}(x,z,t) = A(x,z)\exp(i\omega t - ikz) + c.c., \tag{1}$$

where k is the wave number, ω is the frequency, $A(x,z)$ represents the slowly varying envelope which fulfils standard paraxial wave equation

$$\left(\frac{\partial}{\partial z} - \frac{i}{2k}\frac{\partial^2}{\partial x^2}\right)A(x,z) = \frac{ik}{n_0}\Delta n(E)A(x,z), \tag{2}$$

where n_0 describes unperturbed refractive index and x is the direction in which the beam diffracts. We assume the refractive index change proportional to the square of the electric field E

*e-mail: az@ps.pl

$$\Delta n(E) = \frac{1}{2} n_0^3 s E^2, \tag{3}$$

where s is the quadratic electro-optic coefficient. Although the mentioned previously materials greatly differ in their physical properties, the local relation between internal electric field and light intensity for centrosymmetric medium [6] and PRMQW structure [8] can be estimated with the aid of the same formula

$$E = E_0 \frac{I_\infty + I_d}{I + I_d}, \tag{4}$$

where E_0 denotes the value of the externally applied electric field, $I = |A|^2$ is the light intensity and I_∞ is the intensity of the light at infinity ($x \to \pm\infty$). The dependence of the equivalent dark irradiation I_d on material parameters is the only distinction between considered materials, coming from differences in transport models.

Substituting Eqs. (4) and (3) into Eq. (2) and introducing dimensionless variables, $X = x/w_0$, $\xi = z/(kw_0^2)$ and $\Phi = A/I_d^{1/2}$, with an arbitrary scaling parameter w_0, one obtains the following dimensionless equation

$$i\frac{\partial \Phi}{\partial \xi} + \frac{1}{2}\frac{\partial^2 \Phi}{\partial X^2} + \gamma \frac{(1+\Phi_\infty^2)^2}{(1+\Phi^2)^2}\Phi = 0, \tag{5}$$

where

$$\gamma = \frac{k^2 w_0^2 \Delta n_0}{n_0}, \quad \Delta n_0 = \frac{n_0^3 s E_0^2}{2}, \quad \Phi_\infty = \sqrt{\frac{I_\infty}{I_d}}. \tag{6}$$

Because of quadratic relation between the refractive index change and the electric field E, the polarity of biasing field has no influence on the kind of nonlinearity (self-focusing or self-defocusing). Only the sign of quadratic electro-optic coefficient included in γ determines a type of the refractive index change (negative or positive) and therefore a type of possible soliton solution (bright or dark).

3. Bright soliton solution

For the self-focusing nonlinearity the optical beam intensity is expected to vanish at infinity and Eq. (5) takes the form

$$i\frac{\partial \Phi}{\partial \xi} + \frac{1}{2}\frac{\partial^2 \Phi}{\partial X^2} - \gamma \frac{\Phi}{(1+\Phi^2)^2} = 0. \tag{7}$$

We look for a stationary bright soliton solution of Eq. (7) in the standard form

$$\Phi(X,\xi) = u(X)\exp(i\Gamma\xi), \tag{8}$$

where $u(X)$ is the real function and Γ is the soliton propagation constant. For such localized solutions, Eq. (7) can be integrated once, leading to

$$\Gamma = -\frac{\gamma}{1+u_0^2}, \tag{9}$$

$$\left(\frac{du}{dX}\right)^2 = \frac{2\gamma}{(1+u_0^2)} u^2 \left(\frac{u_0^2 - u^2}{1+u^2}\right), \tag{10}$$

where $u_0 = u(X = 0)$. In the consequence of final integration we obtain

$$\tan^{-1}\left(\sqrt{\frac{\rho_0 - \rho}{1+\rho}}\right) + \frac{1}{\sqrt{\rho_0}} \tanh^{-1}\left(\sqrt{\frac{\rho_0 - \rho}{(1+\rho)\rho_0}}\right) = \sqrt{\frac{2\gamma}{1+\rho_0}} X, \tag{11}$$

where $\rho = u^2$. Equation (11) describes a profile of bright soliton propagating in the photorefractive medium exploiting quadratic electro-optic effect. The choice of the dimensionless variables permits to identify the scaling parameter w_0 with the beam radius. Thus, the axial coordinate is normalized to the diffraction length and the field distribution fulfils the condition

$$\rho(X = 1) = \frac{\rho_0}{2}, \tag{12}$$

which substituted in Eq. (11) gives the relation between the peak intensity and γ parameter

$$\gamma = \left[\frac{1}{\sqrt{\rho_0}} \tanh^{-1}\left(\frac{1}{\sqrt{2+\rho_0}}\right) + \tan^{-1}\left(\sqrt{\frac{\rho_0}{2+\rho_0}}\right)\right]^2 \frac{1+\rho_0}{2}. \tag{13}$$

Because γ is the only parameter of Eq. (7), Eq. (13) shown in Fig. 1 may be treated as an existence curve. As it is apparent from Fig. 1, solitons of the same value of γ can be found for two different peak intensities ρ_0. This feature has been denoted as the soliton bistability [11,12]. Moreover, soliton solutions exist only for γ parameter larger then the critical value $\gamma_{crt} \approx 1.393$. For the minimal value of γ, a single soliton solution exists at the peak intensity $\rho_{0min} \approx 1.12$. The critical parameter γ_{crt} is related with a minimum soliton width which is described by

$$w_{\min} = \frac{1}{kn_0 E_0}\sqrt{\frac{2\gamma_{crt}}{s}}. \tag{14}$$

One of the primary parameters, measured during the solitons experiments is the full width at half maximum (FWHM) of the soliton intensity. In the considered case, FWHM has an analytical form

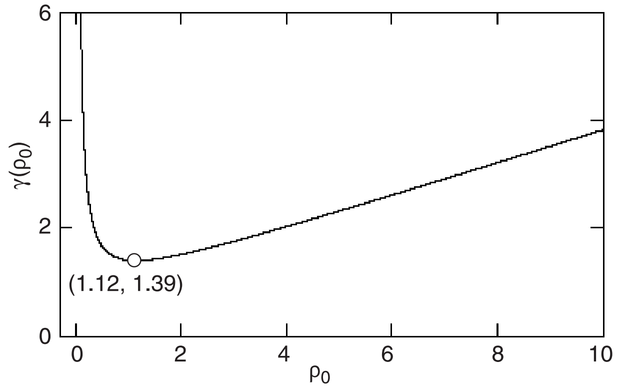

Fig. 1. Relation between the nonlinear parameter γ and peak intensity (existence curve) for bright solitons.

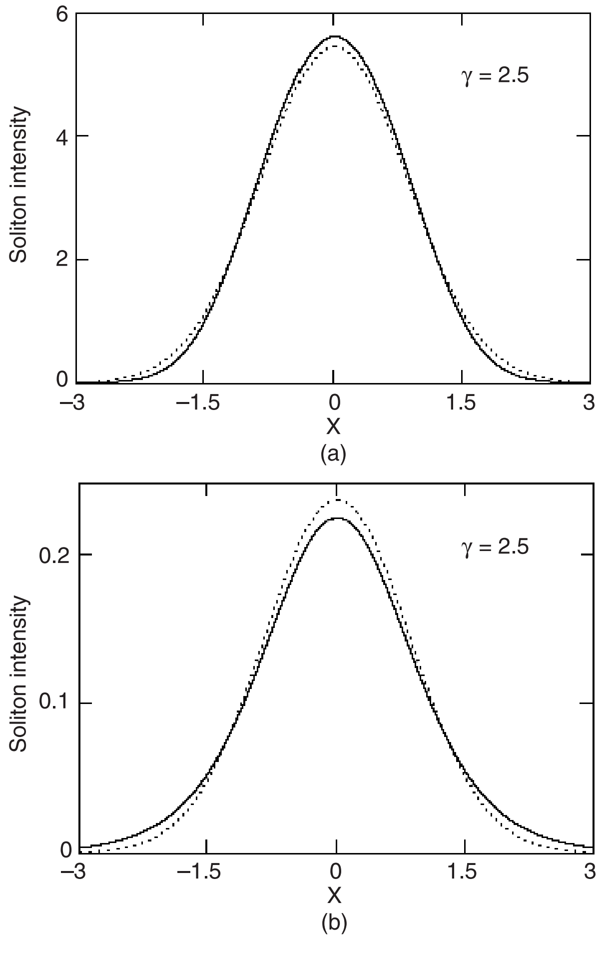

Fig. 2. Comparison between soliton profiles (solid curves) of high intensity (a) and low intensity solution (b) for $\gamma = 2.5$ with the corresponding Gaussian shapes of the same power (dotted curves).

$$FWHM = \frac{1}{k}\sqrt{\frac{2n_0(1+\rho_0)}{\Delta n_0}} \times\left[\frac{1}{\sqrt{\rho_0}}\tanh^{-1}\left(\frac{1}{\sqrt{2+\rho_0}}\right)+\tan^{-1}\left(\sqrt{\frac{\rho_0}{2+\rho_0}}\right)\right]. \quad (15)$$

which directly follows from the definition of γ, Eq. (6).

In Fig. 2, two soliton profiles of high and low intensity for $\gamma = 2.5$ are depicted. Dotted curves in this figure represent Gaussian shapes with the same powers as corresponding solitons.

The soliton power, defined as

$$P = \int_{-\infty}^{\infty} \rho(x)dx, \quad (16a)$$

can be found from Eq. (10) in form

$$P(\rho_0) = \sqrt{\frac{1+\rho_0}{2\gamma}}\left[\sqrt{\rho_0} + (1+\rho_0)\tan^{-1}(\sqrt{\rho_0})\right] \quad (16b)$$

As it can be seen, high intensity solution has higher peak intensity and low intensity solution has lower peak intensity than the corresponding Gaussian shapes. The results are similar to those presented in Ref. 9, for conventional (exploiting linear electro-optic effect) photorefractive medium.

With help of Eqs. (16b) and (9), a simple proof for the stability of considered soliton solution can be performed. It is known that bright solitons are stable if $dP/d\Gamma > 0$ and unstable otherwise [13]. In Fig. 3, the dependence of the soliton power on the propagation constant is plotted. As one can see, the power is a monotonically increasing function of the propagation constant, indicating that the solitons are stable.

4. Beam evolution

4.1. Influence of the initial shape on the beam propagation

The evolution of a beam shape has a fundamental importance for practical implementation. It is well known, that in Kerr media 1D beams with arbitrary initial shape and sufficient power converge to soliton solutions. However, materials exploiting different type of nonlinearity can show different behaviour. Thus, taking advantage of beam propagation method and the results of sec. 3, we studied the evolution of soliton and Gaussian (laserlike) beams in considered medium.

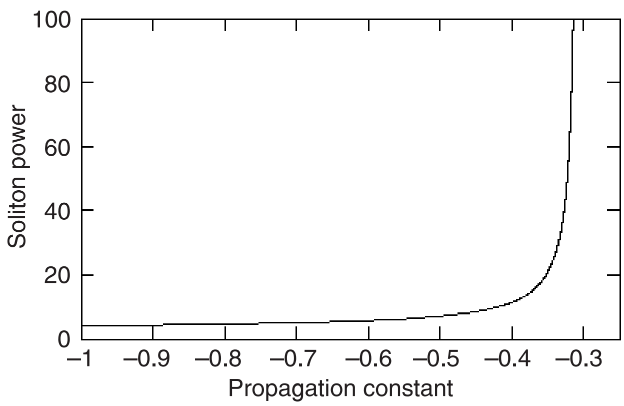

Fig. 3. Soliton power P as a function of propagation constant Γ.

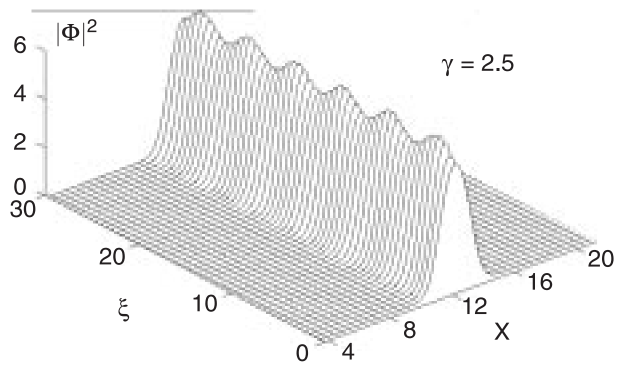

Fig. 4. Evolution of a beam with Gaussian input shape and the same power as the high intensity soliton solution for $\gamma = 2.5$.

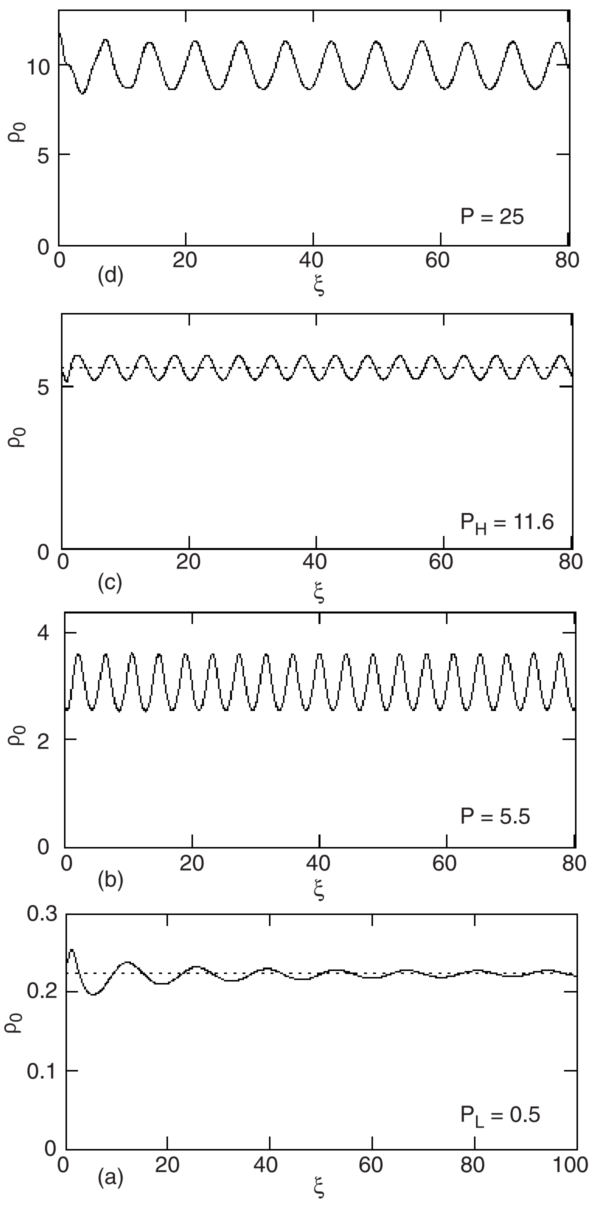

Fig 5. Oscillating amplitude of a beam with Gaussian input shape (solid curves) and steady-state soliton behavior (doted curves) for $\gamma = 2.5$ and different powers: (a) power of low intensity solution $P_L = 0.5$, (b) $P = 5.5$, (c) power of high intensity solution $P_H = 11.6$, and (d) $P = 25$.

First, using finite difference method we integrated Eq. (7) for the exact soliton solution as an initial shape and checked its evolution. As expected, the beam parameters as amplitude and width, remained invariant with propagation distance. Subsequently, we studied the evolution of Gaussian beam with the soliton power obtained for $\gamma = 2.5$, the result for high intensity solution is depicted in Fig. 4. As it can be seen, the beam parameters do not remain invariant with propagation distance. Beam does not converge to a perfect soliton as for Kerr nonlinearity, but shows small oscillation around the soliton parameters. This recurrent motion occurs both in high and low intensity case, as well as for pulses with different power. Figure 5 shows these oscillations for $\gamma = 2.5$ and four different powers. In Fig. 5(a), the oscillating amplitude $\rho_0(\xi)$ of the initial Gaussian beam with the power of low intensity solution ($P_L = 0.5$) is depicted, Fig. 5(c) presents similar dependence for the power of high intensity case ($P_H = 11.6$). As it can be seen, in low intensity case, amplitude of the oscillations decreases but in spite of long propagation distance we did not notice perfect converge to a soliton shape. The smaller amplitude and decaying of oscillations are connected with the fact, that the amplitude of low intensity solution for the considered γ is close to the region described by Kerr nonlinearity. Figures 5(b) and 5(d) show respectively, propagation of a Gaussian beam with the power $P = 5.5$, which is between low and high intensity solutions and evolution of a Gaussian beam with the power $P = 25$, above high intensity solution.

Although, in simulations performed for a moderate value of $\gamma = 2.5$, the beams change their form periodically, in case of the larger γ more complicated recurrent behaviour can occur. In Fig. 6, the evolution of Gaussian beams with power of high intensity solution for several values of γ is presented. It is clearly seen that optical beams with the higher values of γ [Figs. 6(b) and (c)] exhibit more complicated oscillations with more than one characteristic period. We found that such quasi-periodic oscillations of Gaussian beams appear for all high intensity cases with γ value larger then about 4.

The results presented in this section permit to separate three regions of bright soliton intensities in which the evolution of optical beams proceeds in different manner. Starting with low intensities and high values of γ on the existence curve (Fig. 1), we have in sequence: region of low intensities, where arbitrary initial distribution converges to a soliton behaviour like for Kerr nonlinearity, range of moderate intensities and the moderate γ, where arbitrary initial shape leads to a periodic oscillation, and finally region of high intensities where deviation from soliton profile leads to quasi-periodic evolution of optical beams.

It is noteworthy, that oscillating behaviour of solitons has been investigated analytically in the framework of the concept of the soliton internal mode [14]. Furthermore, long-lived oscillating states have been presented in numerical simulations of the dynamics of optical beams in several different systems such as nonlinear quadratic media [15,16]

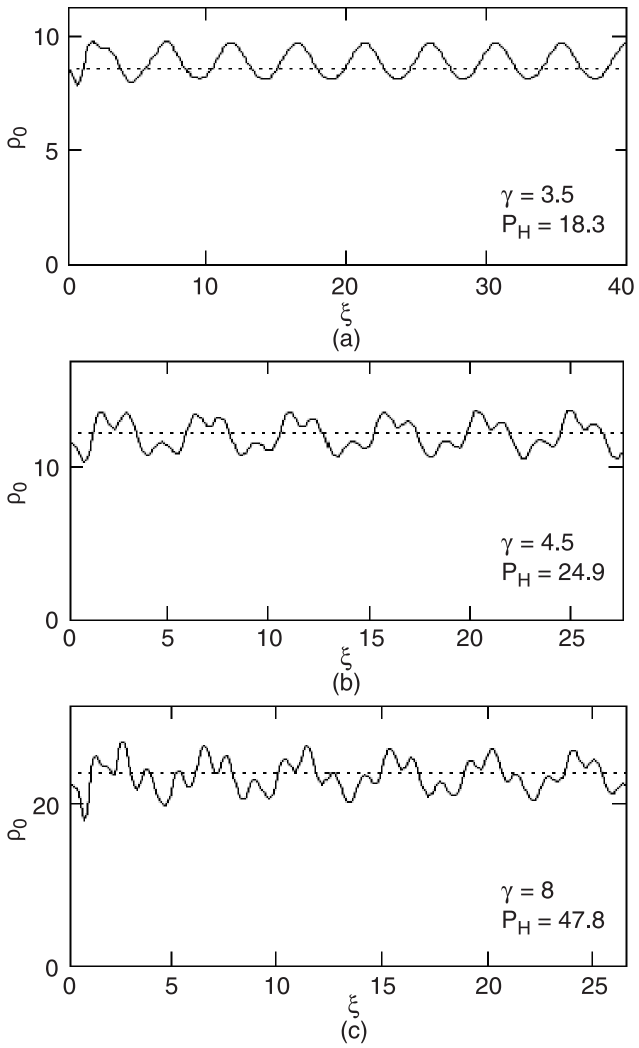

Fig. 6. Evolution of the amplitudes of initially Gaussian beams (solid curves) with power of high intensity soliton solution for several values of γ. Dotted lines mark amplitudes of soliton solutions.

and medium characterized by ideal saturation (threshold nonlinearity) [17].

4.2. Influence of linear losses on soliton propagation

The light absorption is one of the processes intrinsic for the photorefractive effect, thus absorption losses play a significant role in the dynamics of photorefractive solitons. The influence of losses on the evolution of optical beams can be taken into consideration by addition of a loss term to propagation Eq. (7)

$$i\frac{\partial \Phi}{\partial \xi}+\frac{1}{2}\frac{\partial^2 \Phi}{\partial X^2}-\gamma\frac{\Phi}{(1+\Phi^2)^2}=-i\Omega\Phi, \qquad (17)$$

where $\Omega = (\alpha/2)kw_0^2$ and α denotes an absorption coefficient of the medium. We examined the influence of loss on the evolution of bright solitons by numerical integration of Eq. (17) for $\Omega = 0.1$ (a value of this order, e.g., occurs for 15 μm wide soliton in PRMQW with parameters $n_0 = 3.5$, $\lambda = 860$ nm, $\alpha \approx 1$ cm^{-1}, taken from Ref. 18). As an initial distribution, a high intensity soliton solution for $\gamma = 2.3$ and peak intensity $\rho_0 = 5$ was used. As expected, the soliton starts to evolve with the peak amplitude and beam width decreasing as long as it does not reach the minimum on the existence curve. For the peak intensities lower than $\rho_0(\gamma_{crt})$ (which correspond to the minimum on the curve in Fig. 1) soliton width increases. We depicted this evolution in Fig. 7, where the circle marks, inserted on the existence curve, describe the modified values of γ obtained by numerical tracking of the soliton width and amplitude during the propagation with losses.

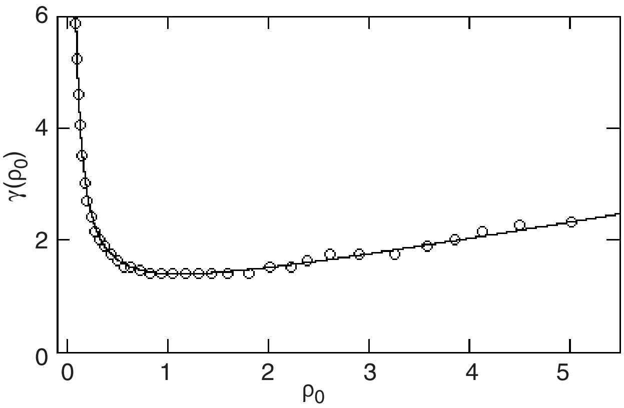

Fig. 7. Existence curve (solid line) and circle marks describing adiabatic change of γ, obtained from numerical integration of Eq. (17).

As it can be seen from Fig. 7, moderate absorption does not destroy soliton behaviour and permits adiabatic evolution along the existence curve. Moreover, we found, that in the initial phase of propagation, both high and low intensity solitons (for $\gamma = 2.3$) try to retain the shape and their width remains unchanged as long as the normalized axial coordinate does not exceed $\xi = 2$ (which corresponds to the actual distance $z = 0.29$ cm for mentioned previously PRMQW). In the next phase of propagation the high intensity beam starts to narrow while the low intensity one begins to widen.

5. Conclusions

We analysed the propagation of light beams in biased photorefractive media with quadratic electro-optic effect. The analytic solution for one-dimensional, bright spatial solitons was found. The soliton solutions are stable and robust against moderate losses. The numerical investigation of Gaussian beams evolution shows that an arbitrary input shape does not converge to a perfect soliton but evolves in periodic or quasi-periodic manner. Similar propagation properties have been found by using different forms of saturable nonlinearity, however, an exact analytic solution of propagation problem presented here, creates possibilities for further theoretical investigation.

Acknowledgements

This work was partially supported by the Polish Committee for Scientific Research (KBN) under research project No. 3 T11B 076 26 accomplished in the years 2004–2006.

References

1. Y. Baek, R. Schiek, G.I. Stegeman, and W. Sohler, ”Interactions between one-dimensional quadratic solitons”, *Opt. Lett.* **22**, 1550 (1997).
2. J.E. Bjorkholm and A. Ashkin, “Cw self-focusing and self-trapping of light in sodium vapour”, *Phys. Rev. Lett.* **32**, 129 (1974); V. Tikhonenko, J. Christou, B. Luther-Davies, “Three dimensional bright spatial soliton collision and fusion in a saturable nonlinear medium**”,** *Phys. Rev. Lett.* **76**, 2698 (1996).
3. M. Segev, G.C. Valley, B. Crosignani, P. DiPorto, and A. Yariv, “Steady-state spatial screening solitons in photorefractive materials with external applied field”, *Phys. Rev. Lett.* **73**, 3211 (1994).
4. D.N. Christodoulides and M.I. Carvalho, “Bright, dark, and gray spatial soliton states in photorefractive media”, J. *Opt. Soc. Am.* **B12**, 1628 (1995).
5. W. Królikowski and B. Luther-Davies, “Analytic solution for soliton propagation in a nonlinear saturable medium”, *Opt. Lett.* **17**, 1414 (1992).
6. M. Segev and A.J. Agranat, “Spatial solitons in centrosymmetric photorefractive media”, *Opt. Lett.* **22**, 1299 (1997).
7. E. DelRe, B. Crosignani, Tamburrini, M. Segev, M. Mitchell, E. Refaeli, and A.J. Agranat, “One-dimensional steady-state photorefractive spatial solitons in centrosymmetric paraelectric potassium lithium tantalite niobate”, *Opt. Lett.* **23**, 421 (1998).
8. A. Ziółkowski and E. Weinert-Rączka, ”Dark solitary waves in photorefractive multiple quantum well planar waveguide“, in *Nonlinear Guided Waves and Their Applications* on CD-ROM, (The Optical Society of America, Washington, DC, 2004), MC 47.
9. S.R. Singh and D.N. Christodoulides, “Evolution of spatial optical solitons in biased photorefractive media under steady state conditions”, *Opt. Comm.* **118**, 569 (1995).
10. S. Gatz and J. Herrmann, “Propagation of optical beams and the properties of two-dimensional spatial solitons in media with a local saturable nonlinear refractive index”, *J. Opt. Soc. Am.* **B14**, 1795 (1997).
11. S. Gatz and J. Herrmann, “Soliton propagation in materials with saturable nonlinearity”, *J. Opt. Soc. Am.* **B8**, 2296 (1991).
12. S. Gatz and J. Herrmann, “Soliton propagation and soliton collision in double-doped fibers with a non-Kerr-like nonlinear refractive-index change”, *Opt. Lett.* **17**, 484 (1992).
13. N.G. Vakhitov and A.A. Kolokolov, ”Stability of stationary solutions of nonlinear wave equations”, *Radiophys. Quantum Electron.* **16**, 783 (1975).
14. Y.S. Kivshar, D.E. Pelinovsky, T. Cretegny, and M. Peyrard, “Internal modes of solitary waves”, *Phys. Rev. Lett.* **80**, 5032 (1998).
15. D.E. Pelinovsky, A.V. Buryak, and Yu.S. Kivshar, “Instability of solitons governed by quadratic nonlinearities“, *Phys. Rev. Lett.* **75**, 591 (1995).
16. C. Etrich, U. Peschel, F. Lederer, B.A. Malomed, and Yu.S. Kivshar, ”Origin of the persistent oscillations of solitary waves in nonlinear quadratic media”, *Phys. Rev.* **E54** 4321 (1996).
17. A.W. Snyder, S. Hewlett, and D.J. Mitchell, “Periodic solitons in optics”, *Phys. Rev.* **E51**, 6297 (1995).
18. E. Weinert-Rączka, M. Wichtowski, A. Ziółkowski, and G. Staroń, ”Photorefractive grating in multiple quantum well planar waveguide”, *Acta Physica Polonica A* **103**, 229 (2003).

