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OPTO-ELECTRONICS REVIEW **14**(3), 179–186

DOI: 10.2478/s11772-006-0023-z

Modelling of isotropic double negative media for microwave applications

Invited papers

I. VENDIK*, O. VENDIK, I. KOLMAKOV, and M. ODIT

Microwave Microelectronics Laboratory, Dept. of Microelectronics & Radio Engineering, St.-Petersburg Electrotechnical University, 5 Prof. Popov Str., 197376, St.-Petersburg, Russia

A composite medium consisting of two sublattices of dielectric spherical particles of high permittivity and different radii embedded in a dielectric matrix of smaller permittivity are considered. It has been shown that such a composite medium reveals properties of an isotropic double negative media (DNG) in a limited frequency range, when resonance oscillations of H_{111} *mode in one kind of particles and* E_{111} *mode in another kind of particles are excited simultaneously. The* E_{111} *resonance and the* H_{111} *resonance give rise to the magnetic dipole momentum and the electric dipole momentum correspondingly. Averaging the magnetic momentum and the electric momentum over the cells belonging to the appropriate spherical particles gives the negative permittivity and permeability. The model of diffraction of a plane electromagnetic wave on a dielectric sphere is presented and compared with the mixing rule based consideration. The results obtained are rather close. Distribution of the electromagnetic wave outside the sphere is calculated. Influence of the dispersion of the sphere size and the dielectric permittivity on the effective parameters of the DNG material is estimated.*

Keywords: composite medium, double negative, metamaterial, spherical particle, mixing rule, diffraction, isotropic.

1. Introduction

Media with simultaneously negative permittivity and permeability or, so-called, double negative media (DNG) is under relentless interest of physicists and microwave engineers [1–3]. The most of practically realized DNG structures, which are also known as left-handed metamaterials, are anisotropic [2] whereas for some applications isotropic material is required. Analysis of negative permittivity and permeability of an isotropic medium formed by a lattice of perfectly conducted particles was performed in Ref. 4. The isotropic three-dimensional left-handed metamaterials based on a symmetrical configuration of a unit cell with split-ring resonators and wires was suggested and analysed in Ref. 5. In Ref. 6, 3D isotropic magnetic metamaterial with a single cell made of six planar resonators with 90° rotational symmetry, placed on the faces of a dielectric cube, was presented. It was confirmed by experimental investigations that the structure behaved as an isotropic magnetic material.

The first model of the isotropic DNG material consisting of small isotropic spheres regularly situated in a dielectric background was introduced by Holloway and Kuester [7]. In this paper, the use of two sets of spherical particles is proposed. In one set, the spheres are made of a high-permittivity dielectric, and in the other set, the spheres are made of a high-permeability magnetic material. The resonant dielectric spheres provide effective negative permittivity, and the resonant magnetic spheres provide effective negative permeability.

*e-mail: IBVendik@eltech.ru

More interesting isotropic structure suitable for practical realization was introduced in Refs. 8 and 9. It was suggested that the artificial material is composed of two sets of dielectric spheres embedded in a host dielectric material. The spheres are made from the same dielectric material and have different radii. The dielectric constant of the spherical particles is much larger than that of the host material. The wavelength inside the sphere is comparable with the diameter of the sphere and, at the same time, the wavelength outside the sphere is large as compared with the sphere dimensions. By combining two sets of the spheres with suitable radii, different modes can be simultaneously exited in the spheres, the magnetic resonance mode in one set of the spheres and the electric resonance mode in the other set.

The previous theoretical consideration of the system suggested was based on the known electrodynamics of a composite medium [7,10]. A medium composed of a periodic lattice of spherical particles considered as scatterers, generates dielectric polarization and magnetization according to the distribution of the scatterers and their polarizabilities. A mixture consisting of an array of the scatterers embedded in a host media is effective medium relative to the propagating wave. When the size of the spherical scatterers is small as compared to the wavelength in the host material and is not small in the material of the scatterers, the effective medium parameters become frequency-dependent. The modelling of electromagnetic response of spherical inclusions embedded in a host material is based on the generalized Lewin's model [10], where the spherical particles with the radius a are arranged in a cubic lattice with the lattice constant s. The incident electromagnetic plane wave propagating in the host material excites the certain modes in the particles. These modes are not strongly eigenmodes of spherical dielectric resonator but they can be specified as H or E modes at the frequencies which are close to the spherical cavity eigenfrequencies. The relative effective permittivity ε_{eff} and the effective permeability μ_{eff} were calculated in Refs. 8 and 9 and compared with the results of full-wave analysis.

The main goal of this paper is to present a theoretical description of the bi-spherical artificial DNG metamaterial based on the theory of diffraction. When the resonant frequency of E_{111} mode in a bigger sphere coincides with the resonant frequency of H_{111} mode in a smaller sphere, the DNG response is expected. The E_{111} resonance gives rise to the magnetic dipole momentum of the spherical particle and the H_{111} resonance gives rise to the electric dipole momentum. Averaging the magnetic momentum and the electric momentum over the cells belonging to the appropriate spherical particles reveals the negative permeability and the permittivity correspondingly. The distance between particles is much smaller than the wavelength in the matrix media. Therefore the averaging momenta mentioned can be found in a quasi-static approach. At the same time, the higher value of the dielectric permittivity of the spherical particle material provides the electromagnetic resonance inside the particles which is described in electrodynamics approach. As a material for manufacturing the spherical particles, one can use dielectrics with the high dielectric constant ($\varepsilon_r > 200$), in particular, ferroelectric single crystals or ceramic samples.

2. Symmetry of bi-spherical DNG structure

Let us consider two sets of the spherical particles arranged in the NaCl structure (Fig. 1). This structure is a member of the cubic system of symmetry and pertains to the class $m3m$. In the case of cubic symmetry, the second rank tensors of all physical parameters of the media are diagonal and characterized by the components of the same values [11,12]. Thus, the permittivity and permeability tensors have the following forms

$$\varepsilon = \begin{vmatrix} \varepsilon_{eff} & 0 & 0 \\ 0 & \varepsilon_{eff} & 0 \\ 0 & 0 & \varepsilon_{eff} \end{vmatrix}, \quad \mu = \begin{vmatrix} \mu_{eff} & 0 & 0 \\ 0 & \mu_{eff} & 0 \\ 0 & 0 & \mu_{eff} \end{vmatrix}, \qquad (1)$$

where the sub-indices eff are introduced to stress that the permittivity and permeability are obtained as a result of averaging electric and magnetic polarization of spherical particles embedded in the matrix. Body-centred and face-centred structures are characterized by the same forms of the second order tensor as the simple cubic structure [11]. For averaging of the polarization of spherical particles embedded in the matrix, one needs to find the volume of the matrix falling on each particle considered. The rigorous evaluation of this volume in the crystal lattice is given by the volume of the Wigner-Seitz cell [12]. For the lattice of cubic symmetry, the volume of the Wigner-Seitz cell can be evaluated as s^3, where s is the distance between the nearest neighbours of the two-component "crystal lattice" (Fig. 1).

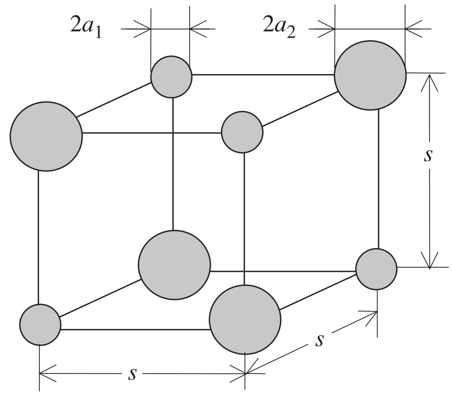

Fig. 1. A periodic composite medium consisting of two sub-lattices of dielectric spherical particles with different radii embedded in a host.

In the case of ferroelectric spherical particles made from perovskite single crystal belonging to the cubic symmetry system, or from ceramic samples, the symmetry of the spherical particles does not influence the symmetry of the permittivity or permeability tensor of the DNG media.

We should stress that the isotropy of the media considered is valid only for the second rank tensors. If one considers the phenomena like dielectric nonlinearity or electrostriction, which are described by fourth rank tensors, the specific anisotropy of media formed by the embedded spherical particles should be taken into account.

3. Electromagnetic field diffraction on dielectric spherical particles

Different analytical models for the double negative (DNG) medium description were introduced to describe the structure with the sets of spherical particles [7–10,13]. The modelling of electromagnetic response of spherical inclusions embedded in a host material is based on the generalized Lewin's model [10]. Originally, the Lewin's model has been specified only for spherical particles with the same radius a arranged in a cubic lattice with the lattice constant s. The spheres are assumed to resonate either in the first or second resonance mode of the Mie theory [14]. Expansion of the model for the case of two sub-lattices of dielectric spherical particles with different radii makes possible to describe the DNG media [8,9]. The properties of DNG media required can be observed in the frequency region, where the resonance of the E- mode in one set of particles and the resonance of the H- mode in another set of particles are excited simultaneously. The improved model of the bi-spherical structure was presented in Ref. 13. The effective permittivity ε_{eff} for a material with two types of inclusions having two different electric polarizabilities was calculated from the generalized Claussius-Mossotti relation [13,14] taking into account the electric polarizabilities of spheres in the magnetic resonance and in the electric resonance mode. Important is a consideration of remaining static electric polarizability of spheres in the magnetic resonance modes, which is not equal to zero as in Refs. 8 and 9.

The numerical analysis of the bi-sphere structure [8] revealed that the interference of the adjacent spherical particles is negligibly small. That makes possible to solve electromagnetic problem for each sphere independently of one another. We consider diffraction of a plane electromagnetic wave on a dielectric sphere. In principle, the problem is solved in the book of Stratton [16]. Some results of solving this problem, as applied to the bi-sphere structure, were presented in Refs. 17 and 18.

Let us consider the diffraction of a plane electromagnetic wave with an amplitude of the electric field E_0 linearly polarized along x- axis. The wave propagates along the z- axis [Fig. 2(a)]

$$\vec{E}(z,t) = \vec{e}_x E_0 e^{i(\omega t - k_2 z)}, \; \vec{H}(z,t) = \vec{e}_y \frac{k_2}{\omega \mu_o} E_0 e^{i(\omega t - k_2 z)}. \quad (2)$$

The wave number k_2 will be defined later.

In order to fulfil the boundary conditions on the surface of the spherical particle, with respect to tangential components of electric and magnetic fields, expansion of the incident plane wave in terms of spherical function is used. The

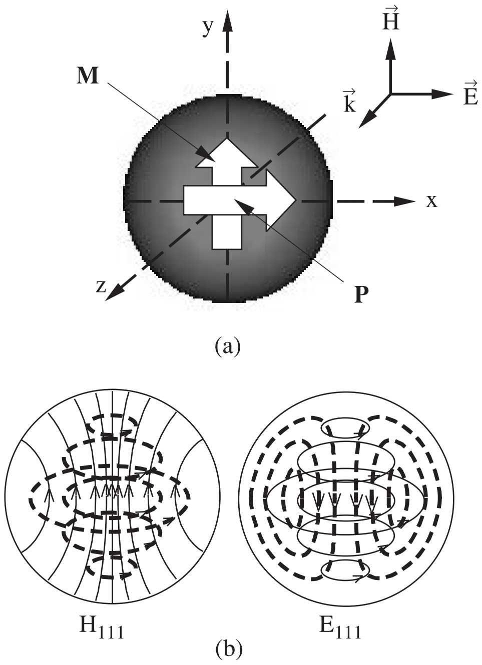

Fig. 2. Spherical particle in the field of linearly polarized electromagnetic wave and field distribution in the equatorial plane: (a) dipole momentum of electric polarization of the particle **P**, and dipole momentum of magnetization of the particle **M** (b) and (b) mode charts of the dominant H_{111} and E_{111} modes in spherical resonator with magnetic walls. Solid and dashed lines show the magnetic and electric field lines, correspondingly.

spherical modes inside the sphere and spherical modes propagating in open space outside the sphere are taken into consideration as well. The boundary conditions give rise to two pairs of non-homogeneous equations with respect to complex amplitudes of the spherical functions inside and outside the spherical particle.

The fields inside the spherical particle are presented in the following form

$$\vec{E}^{(in)} = E_0 e^{i\omega t} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} (a_n^{(in)} \vec{m}_{o1n} - i b_n^{(in)} \vec{n}_{e1n}), \quad (3)$$

$$\vec{H}^{(in)} = -\frac{k_1}{\omega \mu_0} E_0 e^{i\omega t} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} (b_n^{(in)} \vec{m}_{e1n} + i a_n^{(in)} \vec{n}_{o1n}), \quad (4)$$

where $\vec{m}_{o/e,m,n}$ and $\vec{n}_{o/e,m,n}$ are spherical wave functions (odd and even) [16]. As far as the incident wave in open space is linearly polarized, the number $m = 1$ is taken in Eqs. (3) and (4). The wave numbers are defined as

$$k_1 = \omega \sqrt{\varepsilon_0 \varepsilon_1 \mu_0}, \;\; k_2 = \omega \sqrt{\varepsilon_0 \varepsilon_2 \mu_0}, \quad (5)$$

where ε_0 and μ_0 are dielectric permittivity and permeability of free space, ε_1 and ε_2 are the relative permittivities of the spherical particle and matrix material respectively. The diffracted field outside the spheres is given in Ref. 16.

The solutions of the system of equations specified by boundary conditions are resulted for amplitudes of the waves inside the spherical particle in the following form:

- for the waves of magnetic type [$E_r = 0$, Fig. 2(b), left]

$$a_n^{(in)} = -\frac{j_n(\rho)[\rho h_n^{(1)}(\rho)]' - h_n^{(1)}(\rho)[\rho j_n(\rho)]'}{j_n(N\rho)[\rho h_n^{(1)}(\rho)]' - h_n^{(1)}(\rho)[N\rho j_n(N\rho)]'}, \quad (6)$$

- for the waves of electric type [$H_r = 0$, Fig. 2(b), right]

$$b_n^{(in)} = -\frac{j_n(\rho) N[\rho h_n^{(1)}(\rho)]' - h_n^{(1)}(\rho) N[\rho j_n(\rho)]'}{N^2 j_n(N\rho)[\rho h_n^{(1)}(\rho)]' - h_n^{(1)}(\rho)[N\rho j_n(N\rho)]'}, \quad (7)$$

where $\rho = k_2 a$, a is the radius of the spherical particle, $j_n(z)$ is the spherical Bessel function, $h_n^{(1)}(z)$ is the spherical Hankel function of the first order, the sign [ ]′ means the differentiation with respect to ρ or $N\rho$, $N = k_1/k_2$.

Figure 3 represents the distribution of the electromagnetic field components in line with Eqs. (2)–(4) for the spheres with the dielectric permittivity $\varepsilon_p = 400$ surrounded by the air ($\varepsilon_h = 1$). The normalized, to E_0, field components were calculated for the E_{111} mode [Fig. 3(a)] and for the H_{111} mode [Fig. 3(b)]. Diagrams were plotted for values of the polar angles $\Theta = \pi/4$ and $\varphi = \pi/4$. Vertical dashed lines correspond to the boundary of the spherical particles, a_1 is

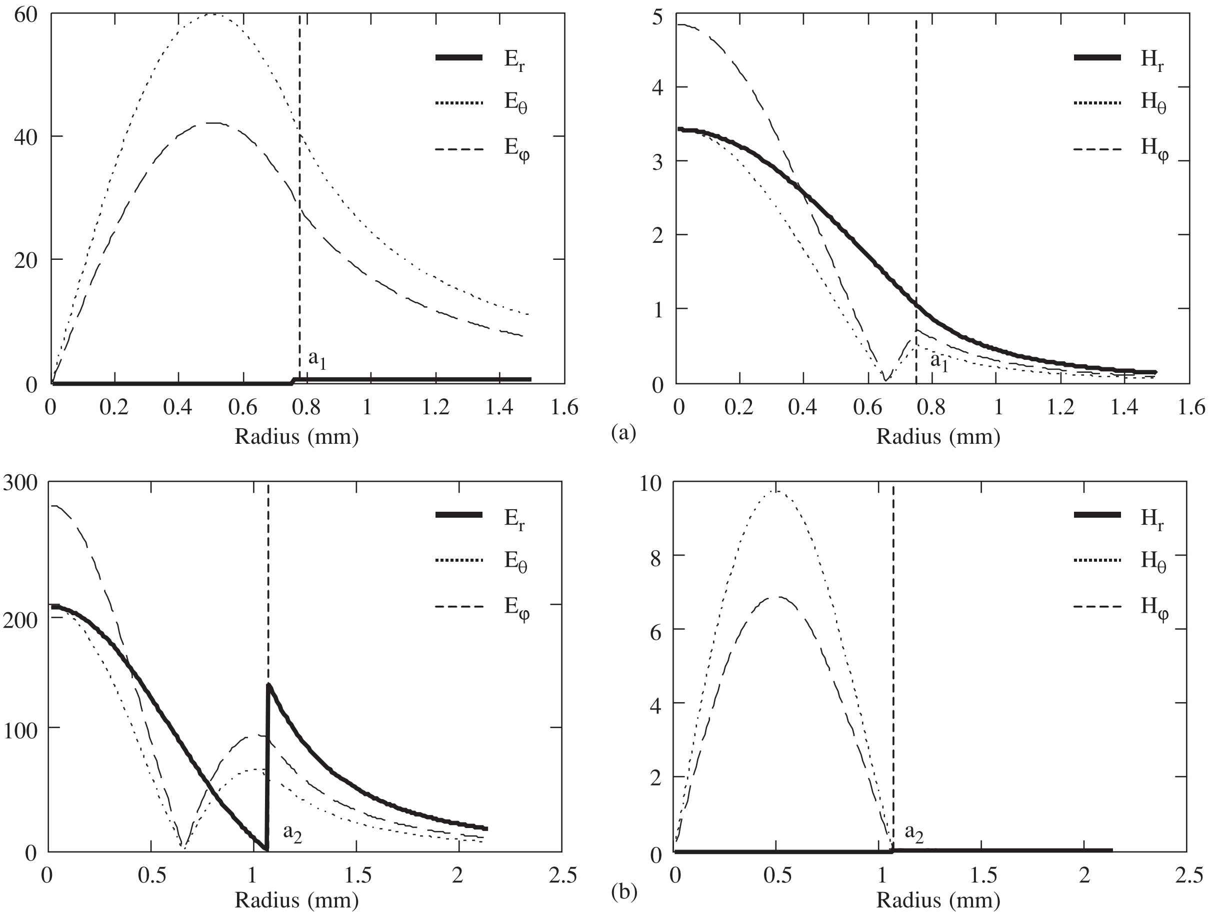

Fig. 3. (a) Electromagnetic field components distribution for magnetic type waves; $E_r = 0$ inside the sphere and (b) electromagnetic field components distribution for electric type waves, $H_r = 0$ inside the sphere.

the radius of the bigger spherical particle and a_2 is the radius of the smaller one. Sizeable electromagnetic field attenuation outside the spheres is observed. Field modules are reduced *e* times at the distance less, than the sphere radius. That means that, if the appropriate distance between the particles is chosen, there is no remarkable electromagnetic interaction between the particles in the structure considered. In some certain points, a graph curve almost reaches zero value. These are the points on a surface inside the sphere where the components of electromagnetic field are close to zero value. This can be explained by the existence of non-perfect magnetic walls on the surface of real boundary of the dielectric spherical particle.

Analysis of Eqs. (6) and (7) is followed by the two important conclusions:

- at certain frequencies, modulus of the denominators of the fractions of Eqs. (6) and (7) become to be minimum what corresponds to the resonance phenomena but because of complex nature of the Hankel functions do not lead to singularities,
- imaginary components of the Hankel functions determine the quality factor of the resonator, which is finite even in the case of lossless material of the spheres. Physically that can be explained by losses caused by a radiation of the diffracted waves outside the sphere.

4. Effective permittivity and permeability of bi-sphere lattice

The spherical particle electric dipole momentum $D_x^{(E)}$oriented along the x axis and magnetic dipole momentum $D_y^{(M)}$oriented along the y axis [Fig. 2(a)] are calculated as follows

$$D_x^{(E)} = \varepsilon_0 \varepsilon_1 \int_{V_{sph}} (\vec{E}^{(in)}(r,\theta,\varphi)\vec{e}_x - E_0 \vec{e}_x) dv, \tag{8}$$

$$D_y^{(M)} = \mu_0 \int_{V_{sph}} \left(\vec{H}^{(in)}(r,\theta,\varphi)\vec{e}_y - \frac{k_1}{\omega \mu_0} E_0 \vec{e}_y \right) dv. \tag{9}$$

While integrating the scalar product of the basis vectors $\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi$ and $\vec{e}_x, \vec{e}_y$ should be taken into account. The averaged macroscopic magnetization and averaged macroscopic electric polarization can be found as the corresponding dipole momentum divided by the volume of the cell containing the dipoles [19]. Thus, one obtains the relative effective permittivity and permeability

$$\varepsilon_r^{(eff)}(\omega) = \frac{D_x^{(E)}(\omega)}{s^3 \varepsilon_0 E_0} + \varepsilon_2, \tag{10}$$

$$\mu_r^{(eff)}(\omega) = \frac{D_x^{(M)}(\omega)}{s^3 E_0 \dfrac{k_2}{\omega}} + 1. \tag{11}$$

After calculation of the integrals in Eqs. 8 and 9, in accordance with Eqs. (3) and (4), where the spherical wave functions should be used [16], one obtains

$$\varepsilon_r^{(eff)}(\omega) = \frac{4}{3}\pi a_2^3 \frac{1}{s^3}\varepsilon_1 b_1^{(in)}(k_1 a_2) I(k_1 a_2), \tag{12}$$

$$\mu_r^{(eff)}(\omega) = \frac{4}{3}\pi a_1^3 \frac{1}{s^3}\sqrt{\varepsilon_1} a_1^{(in)}(k_1 a_1) I(k_1 a_1). \tag{13}$$

Here $I(\xi)$ is the result of integration over the volume of the particle, a_1 and a_2 are the radii of particles, $a_2 > a_1$. The function $I(\xi)$ has been approximated in the region $3 < \xi < 5$ by the following simple formula

$$I(\zeta) = 0.1852(4.5 - \zeta) + 0.0438(4 - \zeta)^2. \tag{14}$$

The frequency dependence of the wave amplitude for the excited modes $a_1^{(in)}$ and $b_1^{(in)}$ determines the frequency dependence of $\varepsilon_r^{(eff)}$ and $\mu_r^{(eff)}(\omega)$. Considering the structure composed by two sub-lattices of the dielectric spherical particles with different radii, we can adjust these radii to obtain the same resonant frequencies for H_{111} mode in the smaller sphere and E_{111} mode in the bigger sphere. Figure 4 presents the simulated frequency dependence of $\varepsilon_r^{(eff)}(\omega)$ and $\mu_r^{(eff)}(\omega)$ for a_1 = 0.748 mm, a_2 = 1.069 mm, s = 4 mm, dielectric permittivity of the particle ε_1 = 400 and $tan\delta = 10^{-3}$, and permittivity of the matrix $\varepsilon_2 = 1$.

One may see that at the frequency slightly above f = 10 GHz, both the permittivity $\varepsilon_r^{(eff)}$ and the permeability $\mu_r^{(eff)}$ are negative. Thus, in the rather narrow frequency band around f = 10 GHz, the existence of isotropic double negative media has been theoretically substantiated. Negative refraction bandwidth depends on the permittivity of the spherical particles. The smaller value of permittivity of dielectric spherical particles, the wider is the frequency range where both effective permittivity and permeability are negative. Negative refraction bandwidth dependence on material constituent particles permittivity is presented in Fig. 5.

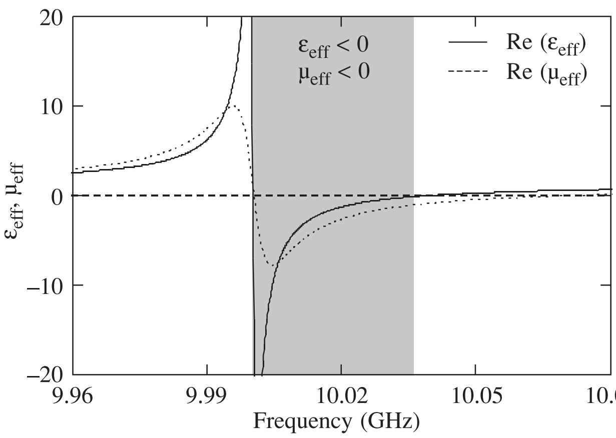

Fig. 4. The effective permittivity and permeability versus frequency f_0 = 10 GHz, ε_p = 400, ε_h = 1, μ_p = 1, μ_h = 1, a_1 = 0.748 mm, a_2 = 1.069 mm, and s = 4 mm.

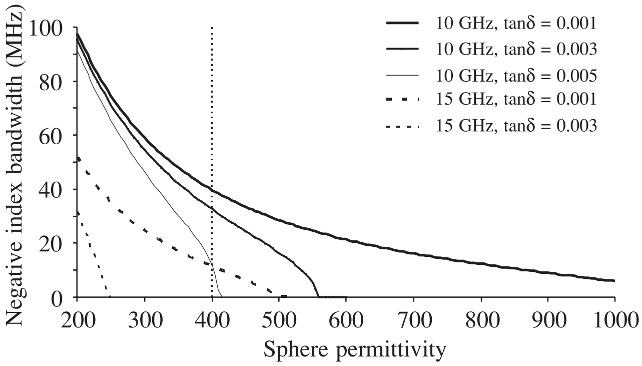

Fig. 5. Dependence of negative refractive index bandwidth on spherical particle permittivity for two resonance frequencies f_1 = 10 GHz, f_2 = 15 GHz and different loss levels.

The Clausius-Mossotti (Maxwell-Garnett) mixing relation and polarizabilities of spheres near the first two Mie resonance modes can be also used to find equations for effective permittivity and permeability as it was done in Ref. 13. In this model, the remaining static electric polarizability of spheres in the magnetic resonance modes was taken into account. The following equations for the effective permittivity and permeability were obtained

$$\frac{\varepsilon_{eff} - \varepsilon_2}{\varepsilon_{eff} + 2\varepsilon_2} = \frac{f_e}{\varepsilon_2}\left(\frac{2\varepsilon_2 + \varepsilon_1 F(\rho_e)}{\varepsilon_2 - \varepsilon_1 F(\rho_e)}\right) + \frac{f_m}{\varepsilon_2}\left(\frac{2\varepsilon_2 + \varepsilon_1 F(\rho_m)}{\varepsilon_2 - \varepsilon_1 F(\rho_m)}\right) \tag{15}$$

$$\frac{\mu_{eff} - 1}{\mu_{eff} + 2} = f_m\left(\frac{2 + F(\rho_m)}{1 - F(\rho_m)}\right), \tag{16}$$

where $\rho_e = k_1 a_2$, and $\rho_m = k_1 a_1$,

$$f_e = \frac{4}{3}\pi a_1^3 \frac{1}{s^3}, \quad f_m = \frac{4}{3}\pi a_2^3 \frac{1}{s^3}, \tag{17}$$

are the volume fractions, and

$$F(\rho) = \frac{2\sin(\rho) - \rho\cos(\rho)}{(\rho^2 - 1)\sin(\rho) - \rho\cos(\rho)}. \tag{18}$$

Let us compare the frequency dependence of the both effective dielectric permittivity and the effective magnetic permeability calculated by using different models. Figures 6 and 7 present an example of effective permittivity and permeability as a function of the frequency for three different analytical models, i.e., Lewin's model [10], improved mixing rule [13] taking into account the electrical polarizability of spheres in the magnetic resonance, and the diffraction model. The parameters of the constituent material are ε_1 = 400, ε_2 = 1, $tan\delta = 10^{-4}$, $\mu_1 = \mu_2 = 1$, a_1 = 0.747 mm, a_2 = 1.069 mm, and s = 4 mm.

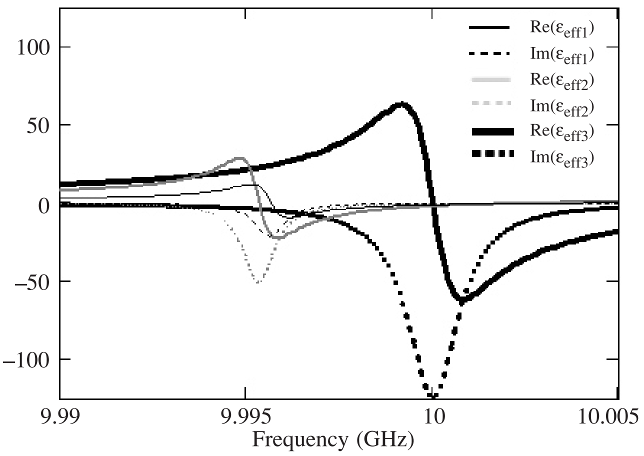

Fig. 6. The effective permittivity as a function of the frequency for three different analytical models, $\varepsilon_{eff\ 1}$ Lewin's model (after Ref. 8), $\varepsilon_{eff\ 2}$ improved mixing rule model (calculated taking into account the electrical polarizability of spheres in the magnetic resonance) (after Ref. 13), and $\varepsilon_{eff\ 3}$ diffraction model. $\varepsilon_1 = 400$, $tan\delta = 10^{-4}$, $\varepsilon_2 = 1$ $\mu_1 = \mu_2 = 1$, $a_1 = 0.748$ mm, $a_2 = 1.069$ mm, and $s = 4$ mm.

The results are in general similar but they differ in the resonant frequency and the magnitude of effective electromagnetic parameters of the medium. The resonant frequency is slightly shifted in comparison with the Lewin's model when the improved mixing equation is used and is shifted more remarkably to the higher frequency for the diffraction model. The structure, containing three single cells with the same characteristics as in legend to Figs. 6 and 7, was simulated by the full-wave analysis. Firstly, structure consisting of four small spheres with the radius $r = 0.748$ mm and then structure consisting of three big spheres with the radius $r = 1.055$ mm, $\varepsilon = 400$, $tan\delta = 10^{-4}$ were simulated. Spheres were placed at the centre of an air-filled rectangular waveguide with the height and width equal to 4 mm, and the length 20 mm [Figs. 8(a) and 8(b)]. Boundary conditions were ideal electric conductor

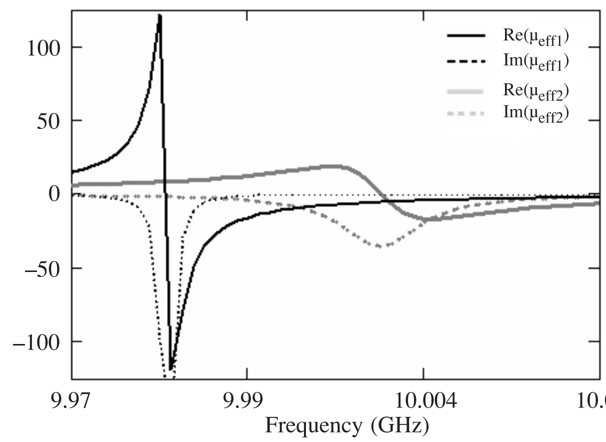

Fig. 7. The effective permeability as a function of the frequency for three different analytical models: $\mu_{eff\ 1}$ mixing rule model (after Refs. 8 and 13), $\mu_{eff\ 2}$ diffraction model $\varepsilon_1 = 400$, $tan\delta = 10^{-4}$, $\varepsilon_2 = 1$ $\mu_1 = \mu_2 = 1$, $a_1 = 0.748$ mm, $a_2 = 1.069$ mm, and s = 4 mm.

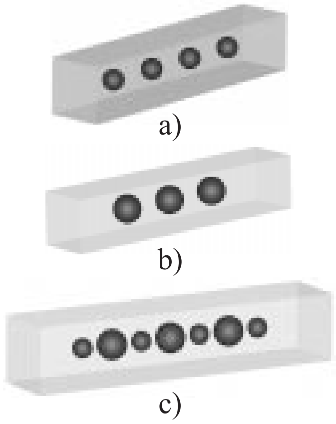

Fig. 8: Three types of simulated medium: four small spheres r = 0.748 mm (a), three big spheres r = 1.055 mm (b), and mixture of both kinds of spheres (c).

(PEC) at the top and bottom of the waveguide and ideal magnetic conductor (PMC) on the sides of the waveguide. The incident electric field was vertically polarized. Then, the structure consisting of both sets of spheres was modelled numerically [Fig. 8(c)]. The results for scattering matrix elements |S11| and |S21| are shown in Fig. 9. It can be seen that there is a stop band around frequency

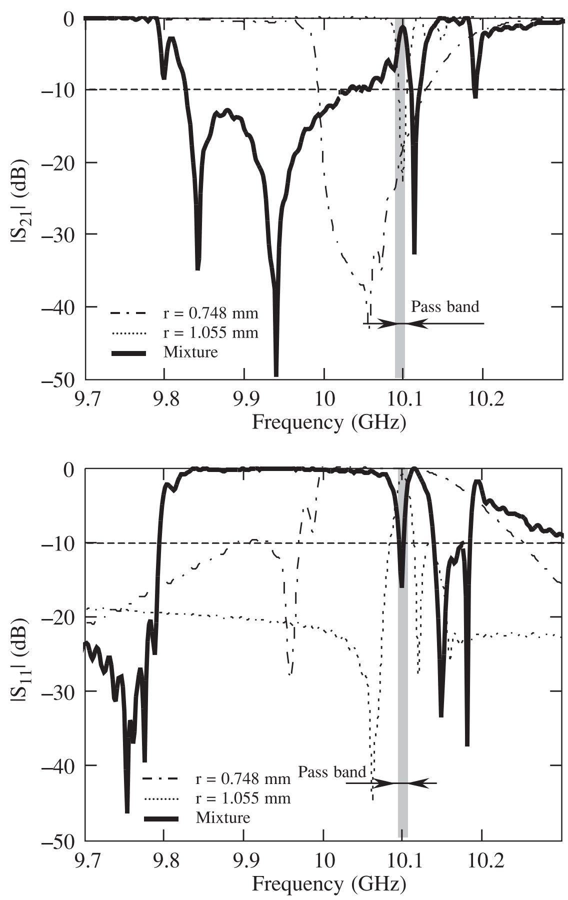

Fig. 9. Numerically calculated S-parameters for a slab consisting of small spheres – dash-dot lines, big spheres – dot lines, and set of both spheres (mixture) – solid line.

10.1 GHz in the case of negative permittivity or permeability only. But, for the medium containing the both sets of spheres there is a narrow pass band near the frequency 10.1 GHz. The frequency range of the electromagnetic wave transmission corresponds to the double-negative characteristics of the structure. Evidently, the resonance frequency is close to that calculated by the diffraction model. Resonance frequency is shifted from 10 GHz to 10.1 GHz. Pass band width has nearly the same value as it was predicted by the diffraction model (Fig. 5). The stop band after pass band arises from the third resonant mode of the larger spheres.

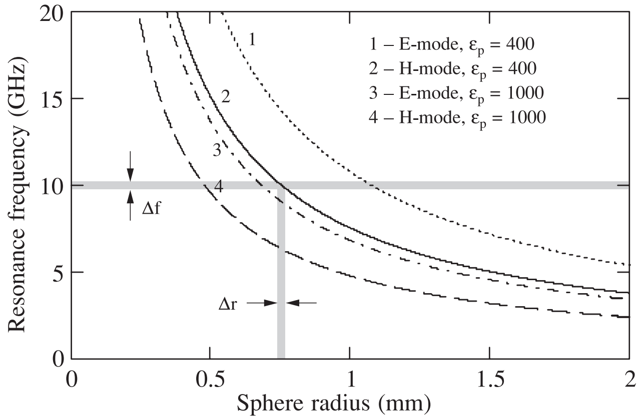

Fig. 10. Dependence of resonance frequency on sphere radius for particles with ε_p = 400 and ε_p = 1000, Δf = 32 MHz, and Δr = 3 μm.

5. Influence of distribution of size and permittivity of spherical particles on DNG characteristics

It has been shown in Refs. 13 and 20 that the statistical distribution of the spherical particle size, caused by production inaccuracy, may affect values of the effective permittivity and permeability of the double negative medium. We can also estimate this dependence using diffraction model. According to Eqs. (12) and (13), the spherical particle radius influences the value of the effective permittivity and permeability.

Dielectric sphere radius variation affects the value of the resonance frequency which corresponds to the low frequency threshold of the negative index range. Let us estimate how the resonant frequency depends on the radius of the constituent spherical particles. The electrical radius of the sphere was defined previously as

$$N\rho = k_1 a, \tag{19}$$

where a is the radius of the particle and $k_1 = \omega\sqrt{\varepsilon_0\varepsilon_1\mu_0}$ is the propagation constant. Let us rewrite Eq. (19) in this way

$$f = \frac{N\rho}{2\pi a\sqrt{\varepsilon_0\varepsilon_1\mu_1}}, \tag{20}$$

where f is the frequency of the electromagnetic wave.

Values of the electrical radius of the resonant spheres can be calculated from Eqs. (6) and (7) for different radius a of the spherical particle. The resonance condition is provided by the minimum of modulus of the denominator in Eqs. (6) and (7). For the given a, the values of electrical radius providing magnetic or electric resonance correspondingly, the resonance frequency can be calculated.

Dependence of the resonant frequency on the sphere radius is shown in Fig. 10. This graph represents the frequency on spherical particle radii dependence for two values of particles permittivity, 400 and 1000. Let us define operational negative index bandwidth as half of full negative bandwidth so that frequency fluctuations in positive and negative directions could be accepted. According to Fig. 5, the negative index bandwidth for DNG medium with spherical inclusions permittivity equal to 400 should be about 16 MHz for 10 GHz resonant frequency. This implies that the spherical particle radius accuracy should be 1.5 μm in this case.

In line with Eq. (20), the resonance frequency is also influenced by the permittivity of the dielectric material of the particles. Figure 11 represents the dependence of the resonant frequency on the spherical particle permittivity for two different values of radius, 1 mm and 0.5 mm. To avoid frequency spreading beyond the negative index bandwidth of 32 MHz, the tolerance of the permittivity of material should be ±0.5%.

Speaking of possibility of practical realization of such artificial metamaterial one should mention that recent technologies allow producing dielectric spheres with the accuracy of about 1 μm. At the same time, the achievable accuracy of permittivity of the dielectric material with ε_r >100 is about 5–20%. Despite this, it is really possible to select samples with needed value of permittivity among a large number of all manufactured samples.

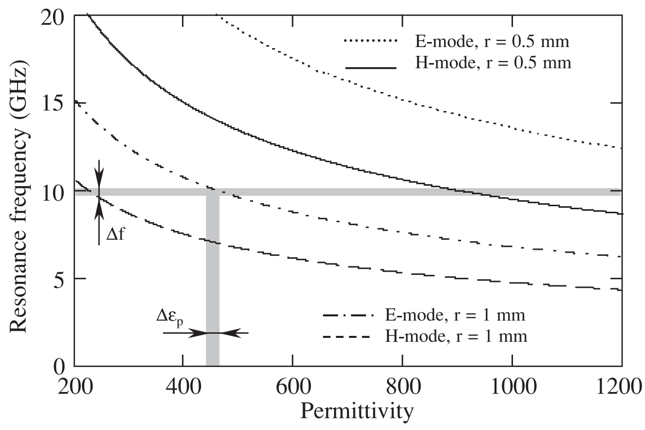

Fig. 11. Dependence of resonance frequency on spherical particle permittivity for particles with r = 0.5 mm and r = 1 mm; Δf = 32 MHz, and $\Delta\varepsilon_p$ = 3.

6. Conclusions

The results obtained show that the composite medium consisting of two sub-lattices of the dielectric spherical particles of high permittivity and different radii embedded in the dielectric matrix of smaller permittivity can be used for a practical realization of the isotropic DNG medium.

A sizeable attenuation of the diffracted electromagnetic wave outside the spherical particle has been shown. That follows by a possibility of neglecting spherical particle mutual interactions, if the appropriate distance between the particles is chosen and by considering each particle as a single independent sphere. In this case, the effective dielectric permittivity and the magnetic permeability can be correctly modelled by a diffraction of the plane wave on the single sphere. Resonant frequency is strongly dependent on spherical particle radius and material permittivity. A very high manufacturing accuracy should be provided to obtain artificial material with the desired properties (negative permittivity and permeability).

Acknowledgements

The work was supported by project No. 500 252 by the NoE "Metamorphose" of 6th FP of EC.

References

1. V.G. Veselago, "Electrodynamics of substrates with simultaneously negative electrical and magnetic permeabilities", *Sov. Phys. Usp.* **10**, 5–13 (1968).
2. D.R. Smith, W.J. Padilla, D.C. Vier, S.C. Nemat-Nasser, and S. Schultz, "A composite media with simultaneously negative permeability and permittivity", *Appl. Phys. Lett.* **84**, 4184–4187 (2000).
3. J.B. Pendry, "Negative refraction makes a perfect lens", *Phys. Rev. Lett.* **85**, 3966–3969 (2000).
4. C.R. Simovski and S. He, "Frequency range and explicit expressions for negative permittivity and permeability for an isotropic medium formed by a lattice of perfectly conducted particles", *Physics Lett.* **A311**, 254–263 (2003).
5. T. Koschny, L. Zhang, and C.M. Soukoulis, "Isotropic three-dimensional left-handed metamaterials", *Phys. Rev.* **B71**, 121103 (R) (2005).
6. L. Jelinek, J.D. Baena, R. Marqués, and J. Zehentner, "How to make a fully isotropic magnetic metamaterial?", *Proc. 12th International Student Seminar on Microwave Applications of Novel Physical Phenomena*, St. Petersburg, 16–18 (2005).
7. C. Holloway and E. Kuester, "A double negative composite medium composed of magnetodielectric spherical particles embedded in a matrix", *IEEE Trans. on AP* **51**, 2596–2603 (2003).
8. O.G. Vendik and M.S. Gashinova, "Artificial duble negative (DNG) media composed by two different dielectric sphere lattices embedded in a dielectric matrix", *Proc. EuMC34*, Paris, France, 1209–1212 (2004).
9. I. Vendik, O. Vendik, M. Gashinova, and I. Kolmakov, "Isotropic artificial double negative media composed by two different dielectric sphere lattices embedded in a dielectric matrix", *Proc. Latsis Symposium*, Lousanne, 108 (2005).
10. L. Lewin, "The electrical constants of a material loaded with spherical particles", *Proc. Inst. Elec. Eng.* **94**, Part III, 65–68 (1947).
11. J.F. Nye, *Physical Properties of Crystals*, Oxford at the Clarendon Press, 1964.
12. N.W. Ashcroft and N.D Mermin, *Solid State Physics*, Holt, Rinehart & Winston, N.Y., 1976.
13. L. Jylhä, I.A. Kolmakov, S. Maslovski, and S.A. Tretyakov, "Modelling of isotropic backward-wave materials composed of resonant spheres", *J. Appl. Phys.* **99**, 043102 (2006).
14. J.A. Kong, *Electromagnetic Wave Theory*, EMW Publishing, Cambridge, Massachusetts, 2005.
15. A. Sihvola, "Electromagnetic mixing formulas and application", *IEE Electromagnetic Waves Series* **47**, 1999.
16. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Co., Inc., 1941.
17. I. Vendik, O. Vendik, and M. Gashinova, "Artificial dielectric medium possessing simultaneously negative permittivity and magnetic permeability", *Tech. Phys. Lett.* **32**, 429–433 (2006).
18. I. Vendik, O. Vendik, O. Mikhail, M. Gashinova, and I. Kolmakov, "Isotropic artificial media with simultaneously negative permittivity and permeability", *Microwave and Optical Techn. Lett.* (to be published)
19. C. Kittel, *Introduction to Solid State Physics*, Wiley, New York, 1976.
20. I.A. Kolmakov, L. Jylhä, S.A. Tretyakov, and S. Maslovski, "Lattice of dielectric particles with double negative response", *28th General Assembly of International Union of Radio Science (URSI)*, New Delhi, India, paper BCD.5 (0109).pdf, 2005.

