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Electromagnetic modelling of 3D periodic structure containing magnetized or polarized ellipsoids

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Coupling matrix and coupling coefficient concepts are applied to the interaction of an incident plane wave with a regular array of small magnetized or polarized ellipsoids, placed in a homogeneous surrounding medium. In general case, the angle of incidence and polarization of the plane wave upon an array of ellipsoids can be arbitrary. In this model, it is assumed that all the ellipsoids are the same, and the direction of their magnetization is also the same. The direction of magnetization is arbitrary with respect to the direction of the propagation of the incident wave and to the boundary plane between the first medium, where the incident wave comes from, and the array material under study. Any magnetized or polarized ellipsoid is represented as a system of three orthogonal elementary magnetic radiators (EMR) and/or three orthogonal elementary electric radiators (EER). Mutual interactions of individual radiators in the array through the incident plane wave and corresponding scattered electromagnetic fields are taken into account. The electrodynamic characteristics – reflection from the surface of the semi-infinite array (in particular, containing uniaxial hexagonal ferrite resonators), transmission through the array, and absorption are analyzed.

Keywords: semi-infinite 3D array, ellipsoidal scatterers, elementary magnetic and electric radiators, hexagonal ferrite resonators, Floquet harmonics, coupling matrix, coupling coefficient, reflection, transmission, absorption.

1. Introduction

The development of microwave and mm-wave technology and engineering of composite media, frequency-selective surfaces, and metamaterials with desirable frequency characteristics for various electromagnetic applications drives an increased interest to behaviour of individual dielectric and/or magnetic particles at their interaction with electromagnetic waves.

In particular, there is an interest in modelling metamaterials and metafilms containing periodic arrays of scatterers with pronounced magnetic properties [1,2]. The application of magnetic particles may give a number of favourable advantages. First, combination of spatial resonances due to periodicity of the structure with ferro-, ferri-, or antiferromagnetic resonances within the scatterers may provide more flexibility in designing the desirable frequency characteristics. Second, the frequency characteristics of such metamaterials can be tuned electronically by applying external bias magnetic field. Third, structures containing magnetic particles may have nonreciprocal electromagnetic properties, especially at frequencies close to their resonances. Fourth, due to the magnetic (spin) nature of resonance phenomena, such metamaterials may be effective absorbers, where surface currents are absent, and corresponding reflection is low. Special attention should be paid to magnetic particles exhibiting natural ferromagnetic resonance, such as particles of uniaxial hexagonal ferrites, since they have high internal field of magnetic crystallographic anisotropy and do not require high bias fields for their resonance operation [3].

The objective of this paper is the development of a simple analytical model of a 3D periodic array of the similar small magnetized or polarized ellipsoids, in particular, uniaxial monocrystalline hexagonal ferrite resonators (FRs).

Interaction of a small crystallographically isotropic ferrite resonator saturated by the external bias magnetic field and placed in a single-mode microwave waveguide was studied long ago [4–7]. One of the approaches to obtain reflection, transmission, and absorption coefficients at the interaction of a FR with the incident electromagnetic mode is the solution of so-called "self-matched field" problem [4]. The small FR is represented as an elementary dipole, excited by the external wave and re-radiating into the waveguide, and the interaction between the FR and the waveguide is then described in terms of a coupling tensor or coupling coefficient [5,8,9]. The coupling depends on the shape, size, and physical properties of a ferrite resonator, mode structure within the waveguide, and the point, i.e., polarization of the microwave magnetic field, where the FR is placed.

In this paper, the concept of the coupling coefficient is applied to the interaction of an incident plane wave with a regular array of small magnetized or polarized ellipsoids, in particular, FRs placed in a homogeneous surrounding medium. In general case, the angle of incidence and polariza-

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tion of the plane wave upon an array of ellipsoids can be arbitrary. In this model, it is assumed that all the ellipsoids are the same, and the direction of their magnetization is also the same. The direction of magnetization is arbitrary with respect to the direction of the propagation of the incident wave and to the boundary plane between the first medium, where the incident wave comes from, and the array material under study. Any magnetized or polarized ellipsoid is represented as a system of three orthogonal elementary magnetic dipoles and/or three orthogonal elementary electric dipoles. Let us call these systems "elementary magnetic radiators" (EMR) and "elementary electric radiators" (EER). Mutual interactions of individual radiators in the array through the incident plane wave and corresponding scattered electromagnetic fields, including inhomogeneous near fields and evanescent modes, are taken into account. The electrodynamic characteristics – reflection from the surface of the semi-infinite array (in particular, containing FRs), transmission through the array, and absorption are analyzed.

It should be mentioned that the description of linear interaction between a plane wave and an array of dipoles is not a new problem. The formulation of such interaction given, for example, in the classical book [10], is based on Ewald's theory of a plane wave reflection from a semi-infinite dipole crystal. However, this theory is not applicable to the dipoles resonant due to their physical nature, such as ferrites. Recently, since the interest to photonic bandgap structures has drastically increased, there are many papers on both analytical and numerical modelling of arrays containing various dielectric or metallic inclusions (see, for example, reviews in Refs. 11 and 12). However, only Ref. 2 contains an analytical solution for plane-wave reflection from a planar interface with a 3D lattice of small magnetized ferrite spheres. This is an approximate solution that neglects polaritons-evanescent waves that appear at the plane boundary of the array. The solution in Ref. 2 is given for a regular 3D structure with the same period along x, y, and z coordinates. The incident wave is along of the coordinate axes, and the ferrite particles are also magnetized along one of the coordinate axes. Recently published paper [13] considers the plane-wave diffraction on semi-infinite photonic (or electromagnetic) crystal with small elements of a general type, and gives the closed-form analytical expressions for the amplitudes of excited modes and scattered spatial Floquet harmonics. The latter form a spectrum of wave vectors for plane waves in a grid of scatterers, widely used in antenna array formulations [14]. In Ref. 13, the interactions between crystal planes are rigorously taken into account through all Floquet harmonics for any distances between the planes. These analytical expressions at the stage of calculating modes of a half-space filled in with FR can be used in our paper as well.

In this paper, we will consider arbitrary orientation of magnetization of FR (or polarization of the corresponding elementary electric radiator) and arbitrary angle of incidence of a plane wave upon a 3D array of scatterers.

2. Fields of individual elementary magnetic and electric radiators

2.1. Elementary magnetic radiator

Suppose that there is a single FR in a homogeneous surrounding space ("base") with the parameters ε_b and μ_b. Since an FR is very small compared to the wavelength in the host medium and periods of the array, its electric polarizability may be neglected compared to the magnetic polarizability, or magnetization (of course, this can be valid only at lower frequencies and frequencies close to the magnetic resonances in a particle). Let represent FR as an EMR, which is a superposition of three independent orthogonal magnetic dipole moments, each of them being equivalent to thin slots in a perfect electric conducting (PEC) screen, or a,

$$\vec{p}^{\,m} = U_x^{eq}\Delta x^{eq}\vec{x}^{\,0} + U_y^{eq}\Delta y^{eq}\vec{y}^{\,0} + U_z^{eq}\Delta z^{eq}\vec{z}^{\,0}. \quad (1)$$

where $U_{x,y,z}^{eq}$ are the equivalent voltages across the corresponding thin slots, and Δx^{eq}, Δy^{eq}, and Δz^{eq} are the corresponding widths of these slots. The magnetic dipole moments in x-, y-, and z- directions are

$$\begin{aligned} p_x^m &= U_x^{eq}\Delta x^{eq} = j\omega\mu_0 m_x \delta(\vec{r}-\vec{r}_0)V_f, \\ p_y^m &= U_y^{eq}\Delta y^{eq} = j\omega\mu_0 m_y \delta(\vec{r}-\vec{r}_0)V_f, \\ p_z^m &= U_z^{eq}\Delta z^{eq} = j\omega\mu_0 m_z \delta(\vec{r}-\vec{r}_0)V_f. \end{aligned} \quad (2)$$

In Eqs. 2, $m_{x,y,z}$ are the components of the microwave (or mm-wave) magnetic moment of the FR. They depend on the external magnetic susceptibility tensor for the FR (see Appendices A and B). V_f is the volume of the FR, $\delta(\vec{r}-\vec{r}_0)$ is the delta-function depending on the coordinates of the point $\vec{r}_0(x_0, y_0, z_0)$, where the FR is placed, and of the point of observation $\vec{r}(x, y, z)$.

Suppose that there is a 3D array of EMR, as shown in Fig. 1. Any EMR has the coordinates $q(x_{lmn}, y_{lmn}, z_{lmn})$, where $x_{lmn} = lD_x$, $y_{lmn} = mD_y$, $z_{lmn} = nD_z$, and indices $l, m, n \in \Im$ are integer numbers varying from $-\infty$ to $+\infty$ (zero indices are for the element in the origin of coordinates). In the particular case of a 2D array of radiators distributed over the plane $z = 0$, the coordinates $z_{lmn} = 0$, and the index $n = 0$ can be omitted. For brevity, let us introduce the notation $\alpha = (lmn)$ for an EMR,

$$\vec{p}_\alpha^{\,m} = p_{x\alpha}^m\vec{x}^{\,0} + p_{y\alpha}^m\vec{y}^{\,0} + p_{z\alpha}^m\vec{z}^{\,0}. \quad (3)$$

The components of electric field re-radiated by this EMR in spherical coordinates are

$$\begin{aligned} E_{r\alpha} &= 0; \\ E_{\theta\alpha} &= \frac{p_{x\alpha}^m \sin\varphi_\alpha - p_{y\alpha}^m \cos\varphi_\alpha}{4\pi} \times \\ &\quad e^{jkR_\alpha}\left(\frac{jk}{R_\alpha} + \frac{1}{R_\alpha^2}\right) \end{aligned} \quad (4)$$

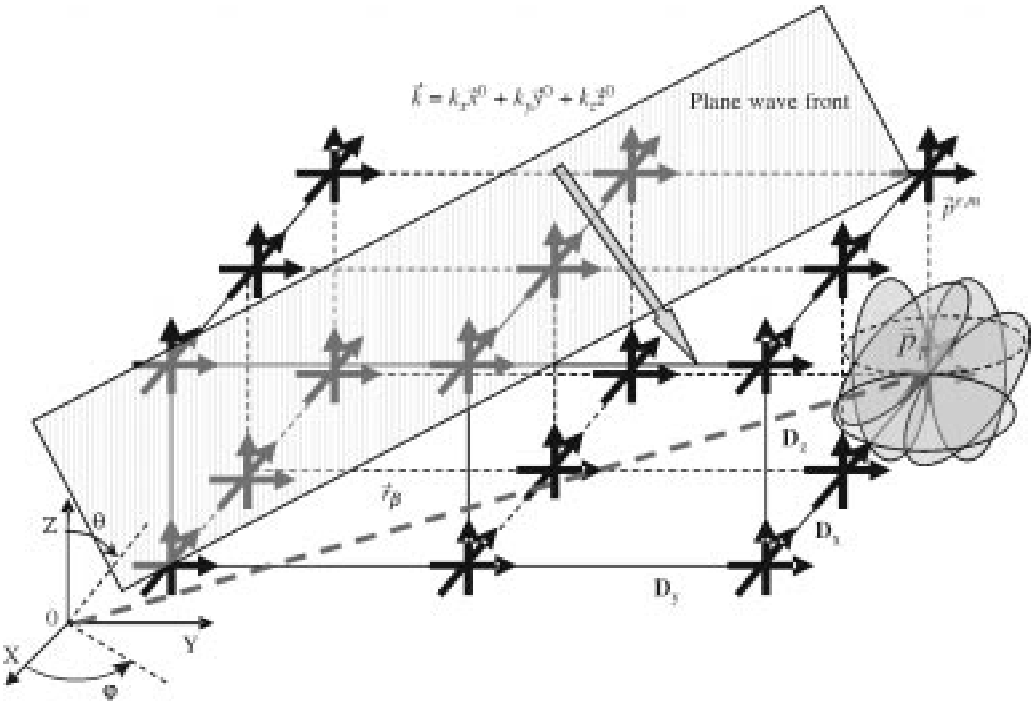

Fig. 1. 3D array of scatterers.

$$E_{\varphi\alpha} = \frac{p^m_{x\alpha}\cos\theta_\alpha\cos\varphi_\alpha + p^m_{y\alpha}\cos\theta_\alpha\sin\varphi_\alpha - p^m_{z\alpha}\sin\theta_\alpha}{4\pi} e^{-jkR_\alpha}\left(\frac{jk}{R_\alpha} + \frac{1}{R_\alpha^2}\right).$$

The components of the magnetic field re-radiated by an individual EMR in spherical system are

$$\begin{aligned}
H_{r\alpha} &= \frac{2(p^m_{x\alpha}\cos\varphi_\alpha + p^m_{y\alpha}\sin\varphi_\alpha)\sin\theta_\alpha - 2p^m_z\cos\theta_\alpha}{4\pi j\omega\mu_b} e^{-jkR_\alpha}\left(\frac{jk}{R_\alpha^2} + \frac{1}{R_\alpha^3}\right),\\
H_{\theta\alpha} &= \frac{(p^m_x\cos\varphi_\alpha + p^m_y\sin\varphi_\alpha)\cos\theta_\alpha - p^m_z\sin\theta_\alpha}{4\pi j\omega\mu_b} e^{-jkR_\alpha}\left(\frac{k^2}{R_\alpha} - \frac{jk}{R_\alpha^2} - \frac{1}{R_\alpha^3}\right),\\
H_{\varphi\alpha} &= \frac{p^m_x\sin\varphi_\alpha - p^m_y\cos\varphi_\alpha}{4\pi j\omega\mu_b}\cos\theta_\alpha e^{-jkR_\alpha}\left(\frac{-k^2}{R_\alpha} + \frac{jk}{R_\alpha^2} + \frac{1}{R_\alpha^3}\right).
\end{aligned} \tag{5}$$

To further consider the 3D regular array of EMR, it is more convenient to work in the Cartesian coordinate system. The unit vectors of the spherical and Cartesian coordinate systems are related as

$$\begin{aligned}
\vec{r}^{\,0} &= \vec{x}^{\,0}\sin\theta\cos\varphi + \vec{y}^{\,0}\sin\theta\sin\varphi + \vec{z}^{\,0}\cos\theta,\\
\vec{\theta}^{\,0} &= \vec{x}^{\,0}\cos\theta\cos\varphi + \vec{y}^{\,0}\cos\theta\sin\varphi - \vec{z}^{\,0}\sin\theta,\\
\vec{\varphi}^{\,0} &= -\vec{x}^{\,0}\sin\varphi + \vec{y}^{\,0}\cos\varphi.
\end{aligned} \tag{6}$$

Then, the electric field components in the Cartesian coordinate system are

$$\begin{aligned}
E_{x\alpha} &= \frac{-(p^m_{y\alpha}\cos\theta_\alpha + p^m_{z\alpha}\sin\theta_\alpha\sin\varphi_\alpha)}{4\pi}\left(\frac{1}{R_\alpha} + \frac{jk}{R_\alpha^2}\right)e^{-jkR_\alpha},\\
E_{y\alpha} &= \frac{(p^m_x\sin\theta_\alpha - p^m_z\sin\theta_\alpha\cos\varphi_\alpha)}{4\pi}\left(\frac{1}{R_\alpha} + \frac{jk}{R_\alpha^2}\right)e^{-jkR_\alpha},\\
E_{z\alpha} &= \frac{(-p^m_x\sin\varphi_\alpha + p^m_y\cos\varphi_\alpha)\sin\theta_\alpha}{4\pi}\left(\frac{1}{R_\alpha} + \frac{jk}{R_\alpha^2}\right)e^{-jkR_\alpha},
\end{aligned} \tag{7}$$

and the corresponding magnetic field components are

$$H_{x\alpha} = \frac{1}{4\pi j\omega\mu_b}\begin{pmatrix} G_{R\alpha}(p^m_{x\alpha}\sin^2\theta_\alpha\cos^2\varphi_\alpha + p^m_{y\alpha}\sin^2\theta_\alpha\sin\varphi_\alpha\cos\varphi_\alpha) + \\ G_{\varphi\alpha}(-p^m_{x\alpha}\sin^2\varphi_\alpha + p^m_{y\alpha}\cos\theta_\alpha\sin\varphi_\alpha) + \\ G_{\theta\alpha}(p^m_{x\alpha}\cos^2\theta_\alpha\cos^2\varphi_\alpha + p^m_{y\alpha}\cos^2\theta_\alpha\sin\varphi_\alpha\cos\varphi_\alpha) - \\ G_{z\alpha}p^m_{z\alpha}\sin\theta_\alpha\cos\theta_\alpha\cos\varphi_\alpha \end{pmatrix},$$

$$H_{y\alpha} = \frac{1}{4\pi j\omega\mu_b}\begin{pmatrix} G_{R\alpha}(p^m_{x\alpha}\sin^2\theta_\alpha\sin\varphi_\alpha\cos\varphi_\alpha + p^m_{y\alpha}\sin^2\theta_\alpha\sin^2\varphi_\alpha) + \\ G_{\varphi\alpha}(p^m_{x\alpha}\sin\varphi_\alpha\cos\varphi_\alpha - p^m_{y\alpha}\cos^2\varphi_\alpha) + \\ G_{\theta\alpha}(p^m_{x\alpha}\cos^2\theta_\alpha\cos\varphi_\alpha\sin\varphi_\alpha + p^m_{y\alpha}\cos^2\theta_\alpha\sin^2\varphi_\alpha) - \\ G_{z\alpha}p^m_z\sin\theta_\alpha\cos\theta_\alpha\sin\varphi_\alpha \end{pmatrix}, \tag{8}$$

$$H_{z\alpha} = \frac{1}{4\pi j\omega\mu_b}\begin{pmatrix} G_{R\alpha}(p^m_{x\alpha}\sin\theta_\alpha\cos\theta_\alpha\cos\varphi_\alpha + p^m_{y\alpha}\sin\theta_\alpha\cos\theta_\alpha\sin\varphi_\alpha + p^m_z\cos^2\theta_\alpha) + \\ G_{\theta\alpha}(-p^m_{x\alpha}\sin\theta_\alpha\cos\theta_\alpha\cos\varphi_\alpha - p^m_{y\alpha}\cos\theta_\alpha\sin\theta_\alpha\sin\varphi_\alpha - p^m_z\sin^2\theta_\alpha) + \end{pmatrix},$$

where the amplitudes for propagating and near-field terms are

$$G_{R\alpha} = e^{-jkR_\alpha}\left(\frac{2jk}{R^2_\alpha} + \frac{2}{R^3_\alpha}\right), \qquad G_{\varphi\alpha} = e^{-jkR_\alpha}\left(\frac{k^2}{R_\alpha} - \frac{jk}{R^2_\alpha} - \frac{1}{R^3_\alpha}\right),$$
$$G_{\theta\alpha} = e^{-jkR_\alpha}\left(-\frac{k^2}{R_\alpha} + \frac{jk}{R^2_\alpha} + \frac{1}{R^3_\alpha}\right), \quad G_{z\alpha} = e^{-jkR_\alpha}\left(-\frac{k^2}{R_\alpha} + \frac{3jk}{R^2_\alpha} + \frac{3}{R^3_\alpha}\right). \tag{9}$$

2.2. Elementary electric radiator

The 3D array of scatterers may be comprised not necessarily of magnetic particles, but dielectric or ferroelectric ones. Also, even when considering magnetic particles, their dielectric properties may be of importance. For these reasons, an individual elementary electric radiator (EER) $\vec{p}^{\,e}_\alpha = p^e_{x\alpha}\vec{x}^{\,0} + p^e_{y\alpha}\vec{y}^{\,0} + p^e_{z\alpha}\vec{z}^{\,0}$ should be considered along with the EMR. The components of its re-radiated electric and magnetic fields can be written analogously to those for the EMR, using the easily proved duality

$$\begin{aligned} &p^m_{x,y} \leftrightarrow -p^e_{x,y}, \\ &\mu_b \leftrightarrow \varepsilon_b, \\ &H \leftrightarrow E. \end{aligned} \tag{10}$$

Thus, in the Cartesian coordinate system, the components of electric field of an individual electric dipole field are

$$E_{x\alpha} = \frac{1}{4\pi j\omega\varepsilon_b}\begin{pmatrix} G_{R\alpha}(p^e_{x\alpha}\sin^2\theta_\alpha\cos^2\varphi_\alpha + p^e_{y\alpha}\sin^2\theta_\alpha\sin\varphi_\alpha\cos\varphi_\alpha) + \\ G_{\varphi\alpha}(-p^e_{x\alpha}\sin^2\varphi_\alpha + p^e_{y\alpha}\cos\theta_\alpha\sin\varphi_\alpha) + \\ G_{\theta\alpha}(p^e_{x\alpha}\cos^2\theta_\alpha\cos^2\varphi_\alpha + p^e_{y\alpha}\cos^2\theta_\alpha\sin\varphi_\alpha\cos\varphi_\alpha) - \\ G_{z\alpha}p^e_{z\alpha}\sin\theta_\alpha\cos\theta_\alpha\cos\varphi_\alpha \end{pmatrix},$$

$$E_{y\alpha} = \frac{1}{4\pi j\omega\varepsilon_b}\begin{pmatrix} G_{R\alpha}(p^e_{x\alpha}\sin^2\theta_\alpha\sin\varphi_\alpha\cos\varphi_\alpha + p^e_{y\alpha}\sin^2\theta_\alpha\sin^2\varphi_\alpha) + \\ G_{\varphi\alpha}(p^e_{x\alpha}\sin\varphi_\alpha\cos\varphi_\alpha - p^e_{y\alpha}\cos^2\varphi_\alpha) + \\ G_{\theta\alpha}(p^e_{x\alpha}\cos^2\theta_\alpha\cos\varphi_\alpha\sin\varphi_\alpha + p^e_{y\alpha}\cos^2\theta_\alpha\sin^2\varphi_\alpha) - \\ G_{z\alpha}p^e_z\sin\theta_\alpha\cos\theta_\alpha\sin\varphi_\alpha \end{pmatrix}, \tag{11}$$

$$E_{z\alpha} = \frac{1}{4\pi j\omega\varepsilon_b}\begin{pmatrix} G_{R\alpha}(p^e_{x\alpha}\sin\theta_\alpha\cos\theta_\alpha\cos\varphi_\alpha + p^e_{y\alpha}\sin\theta_\alpha\cos\theta_\alpha\sin\varphi_\alpha + p^e_z\cos^2\theta_\alpha) + \\ G_{\theta\alpha}(-p^e_{x\alpha}\sin\theta_\alpha\cos\theta_\alpha\cos\varphi_\alpha - p^e_{y\alpha}\cos\theta_\alpha\sin\theta_\alpha\sin\varphi_\alpha - p^e_z\sin^2\theta_\alpha) + \end{pmatrix},$$

with the amplitudes G_α as in Eqs. 9.

The components of magnetic field radiated by an individual EER are

$$H_{x\alpha} = \frac{(p^e_{y\alpha}\cos\theta_\alpha + p^e_{z\alpha}\sin\theta_\alpha \sin\varphi_\alpha)}{4\pi}\left(\frac{1}{R_\alpha} + \frac{jk}{R^2_\alpha}\right)e^{-jkR_\alpha},$$
$$H_{y\alpha} = \frac{(-p^e_y \sin\theta_\alpha + p^e_z \sin\theta_\alpha \cos\varphi_\alpha)}{4\pi}\left(\frac{1}{R_\alpha} + \frac{jk}{R^2_\alpha}\right)e^{-jkR_\alpha}, \tag{12}$$
$$H_{z\alpha} = \frac{(p^e_x \sin\varphi_\alpha - p^e_y \cos\varphi_\alpha)\sin\theta_\alpha}{4\pi}\left(\frac{1}{R_\alpha} + \frac{jk}{R^2_\alpha}\right)e^{-jkR_\alpha}.$$

3. An array of non-interacting EMR

First, let us consider fields radiated by a regular three-dimensional (3D) array of non-interacting EMR in the Cartesian coordinate system. The distance between any point of observation $p(x, y, z)$ and the radiator at the point $q(x_\alpha, y_\alpha, z_\alpha)$ is

$$R_\alpha = \sqrt{(x - x_\alpha)^2 + (y - y_\alpha)^2 + (z - z_\alpha)^2}. \tag{13}$$

Since at this stage the EMR are assumed independent, the total radiated field is the superposition of partial fields (double or triple summation depending on 2D or 3D geometry),

$$\vec{E} = \sum_\alpha \vec{E}_\alpha, \quad \vec{H} = \sum_\alpha \vec{H}_\alpha. \tag{14}$$

Then, the components of the fields of. Eqs. (7) and (8) should be used to find the total radiated field of an array of EMR, and the components of Eqs. (11) and (12) should be used for an array of EER. The angles in Eqs. (7) and (8) and Eqs. (11) and (12) are found from the geometry of the problem,

$$\sin\theta_\alpha = \sin\theta_{lmn} = \frac{\sqrt{(x - lD_x)^2 + (y - mD_y)^2}}{R_{lmn}},$$
$$\cos\theta_\alpha = \cos\theta_{lmn} = \frac{z - nD_z}{R_{lmn}},$$
$$\sin\varphi_\alpha = \sin\varphi_{lmn} = \frac{y - mD_y}{\sqrt{(x - lD_x)^2 + (y - mD_y)^2}}, \tag{15}$$
$$\cos\varphi_\alpha = \cos\varphi_{lmn} = \frac{x - lD_x}{\sqrt{(x - lD_x)^2 + (y - mD_y)^2}}.$$

For a metafilm (2D case), in Eq. (15), the index $n = 0$. It is important to mention that the scatterers in this formulation may be not necessarily all the same. They might have the same parameters from geometrical and material points of view, but their excitation is not in the same phase. There is a delay in excitation of "layers" of FRs along the direction of the wave propagation. Thus, if the incident wave is propagating along the z-axis, the phase shift of the n-th layer excitation compared to the layer in the plane $z = 0$ is $k_z nD_z$, so that $p^m_{x,y,z\alpha} = p^m_{x,y,z} e^{-jk_z nD_z}$. For an arbitrary angle of incidence of the plane wave upon a half-space of FRs, the wave vector is $\vec{k} = k_x \vec{x}^0 + k_y \vec{y}^0 + k_z \vec{z}^0$, and the radius-vector of an individual FR at the point α is $\vec{r}_\alpha = lD_x \vec{x}^0 + mD_y \vec{y}^0 + nD_z \vec{z}^0$. Then the magnetic dipole moment of an FR at the point α is $\vec{p}^m_\alpha = \vec{p}^m_0 e^{-j\vec{k}\vec{r}_\alpha}$.

4. Interactions between individual scatterers of an array

Let the point of observation be moved from an arbitrary point outside the array of scatterers $p(x, y, z)$ to a point where one of the scatterers is situated, for example, $q(x_\beta, y_\beta, z_\beta)$, where index $\beta = (l_1 m_1 n_1)$. Exclude the scatterer from this point. All the other scatterers induce the field in this point determined by the sum of the terms, except for $\alpha = \beta$, where $\alpha = (lmn)$.

$$\vec{H}^{scat}_{\Sigma/\beta} = \sum_{\alpha\neq\beta} \vec{H}_\alpha \quad \text{and} \quad \vec{E}^{scat}_{\Sigma/\beta} = \sum_{\alpha\neq\beta} \vec{E}_\alpha. \tag{16}$$

The distance between any scatterer with the index α and the point of observation β is

$$R_{\alpha\beta} = \sqrt{(l_1 - l)^2 D^2_x + (m_1 - m)^2 D^2_y + (n_1 - n)^2 D^2_z}. \tag{17}$$

To calculate the fields of Eq. (16) induced by the other scatterers in the point of the particular scatterer β, one must substitute $R_\alpha \to R_{\alpha\beta}$ in Eqs. (7) and (8) for EMR, and in Eqs. (11) and (12) for EER.

Along with the incident plane wave, the field due to the other scatterers as in Eq. (16) affects the corresponding electric and magnetic dipole moments of the scatterer placed in the point β. For example, the microwave magnetization and the magnetic "dipole" moment of a ferrite ellipsoidal particle will be calculated as

$$\vec{m}_\beta = \vec{\vec{\chi}}^{ext}_\beta (\vec{h}_\beta + \vec{H}^{scat}_\beta) = \vec{\vec{\chi}}^{eq}_\beta \vec{h}_\beta$$
$$\vec{p}^m_\beta = j\omega\mu_0 V_\beta \vec{m}_\beta \delta(\vec{r} - \vec{r}_\beta) e^{-j\vec{k}\vec{r}_\beta} \tag{18}$$

where, $\vec{h}_\beta = \vec{h}^{inc} + \vec{H}^{scat}_{\Sigma/\beta}$, $\vec{h}^{inc}$ is the magnetic field of the incident plane wave, and $\vec{\vec{\chi}}^{eq}_\beta$ is the equivalent external magnetic susceptibility tensor for any FR, derived in Appendix A. In the general case, the field $\vec{H}^{scat}_{\Sigma/\beta}$ produced by the other EMRs has three components, though the plane-wave field $\vec{h}^{inc}$ might have one or two components, and the total field acting on any individual FR has three components. Then, it is necessary to consider all three components of the vector $\vec{m}_\beta$, and in the general case $\vec{\chi}_\beta^{eq}$ is a nine-component tensor.

If there is an array of radiators, then the total electric and magnetic fields acting upon an individual scatterer at the point β are

$$\vec{e}_\beta = \begin{bmatrix} e_{\beta x}^{inc} + E_{\Sigma/\beta x}^{scat} \\ e_{\beta y}^{inc} + E_{\Sigma/\beta y}^{scat} \\ e_{\beta z}^{inc} + E_{\Sigma/\beta z}^{scat} \end{bmatrix} \text{ and } \vec{h}_\beta = \begin{bmatrix} h_{\beta x}^{inc} + H_{\Sigma/\beta x}^{scat} \\ h_{\beta y}^{inc} + H_{\Sigma/\beta y}^{scat} \\ h_{\beta z}^{inc} + H_{\Sigma/\beta z}^{scat} \end{bmatrix}, \quad (19)$$

where $e_{\beta x,y,z}^{inc}$ and $h_{\beta x,y,z}^{inc}$ are the electric and magnetic field components of the incident wave (in the general case of oblique incidence they depend on polarization and angle of incidence). $E_{\Sigma/\beta x,y,z}^{scat}$ and $H_{\Sigma/\beta x,y,z}^{scat}$ are the fields produced by the other scatterers in the point of observation. The elements of the coupling tensor $\vec{w}_\beta$, relating the field re-radiated by the FR and its magnetization (or polarization in the case of a dielectric resonator) as Eq. (B2) in Appendix B, and represented as Eq. (B12), are calculated through the products of the corresponding magnetic field components (propagation terms $e^{\pm jkR}$ should cancel out). So, the coupling tensor for EMR of the array is

$$\vec{w}_\beta = \begin{bmatrix} w_{11\beta} & w_{12\beta} & w_{13\beta} \\ w_{12\beta} & w_{22\beta} & w_{23\beta} \\ w_{13\beta} & w_{23\beta} & w_{33\beta} \end{bmatrix} = \frac{\omega\mu_0 V_f}{2N_\beta} \begin{bmatrix} h_{x\beta}^2 & h_{y\beta}^2 & h_{z\beta}^2 \\ h_{x\beta}^2 & h_{y\beta}^2 & h_{z\beta}^2 \\ h_{x\beta}^2 & h_{y\beta}^2 & h_{z\beta}^2 \end{bmatrix}. \quad (20)$$

The norm of this total field is found as in Refs. 8 and 9, and it is also discussed in Appendix B,

$$N_\beta = -2\int_S \left\lfloor \vec{e}_\beta \times \vec{h}_\beta^* \right\rfloor d\vec{S} \quad (21)$$

As soon as the coupling tensor $\vec{w}_\beta$ is found for any element β of the array of scatterers, the corresponding equivalent susceptibility tensor $\vec{\chi}_\beta^{eq}$ can be found as Eq. (B13), and then the microwave magnetization and magnetic “dipole” moment Eq. (18) can be easily calculated. They take into account mutual interactions between the FRs in the array. The resultant magnetic “dipole” moments then should be substituted in formulas for calculating the total field produced by the array of elements of Eq. (14). The similar procedure can be done for scatterers represented as EER.

The expression for the transmission T_H and reflection Γ_H coefficients (in terms of magnetic field) can be obtained from the boundary conditions for the fields in the cross-section of interest,

$$T_H \vec{h}^{inc} = \vec{h}^{inc} + \vec{H}_{rad}^{+} \quad (22)$$

and

$$\Gamma_H \vec{h}^{inc} = \vec{H}_{rad}^{-}, \quad (23)$$

where $\vec{h}^{inc}$ is the magnetic field of an incident plane wave. $\vec{H}_{rad}^{\pm}$ are the propagating parts of radiated fields through the bulk of array (positive direction) and backscattered from it (negative direction), calculated through the summation of corresponding scattered fields of individual FRs, expanded in a series of eigenmodes of a semi-infinite 3D array of Eq. (B6) in Appendix B. Similarly, one can get the reflection and transmission coefficients in terms of electric field.

Complex reflection and transmission coefficients for fields are related as

$$T_H = 1 + \Gamma_H. \quad (24)$$

The reflection, transmission, and absorption coefficients in the case of plane waves can be represented in terms of the coupling coefficient η_c, as shown in Eq. (B14), Appendix B,

$$T_H = \frac{1}{1+\eta_c}; \quad (25)$$

$$\Gamma_H = \frac{-\eta_c}{1+\eta_c}. \quad (26)$$

The corresponding absorption coefficient is

$$\alpha_{abs} = \frac{2|\eta_c|}{|1+\eta_c|^2}. \quad (27)$$

It is easy to check that the power balance is always fulfilled,

$$|\Gamma_H|^2 + |T_H|^2 + \alpha_{abs} = 1, \quad (28)$$

where the reflection coefficient in terms of power is

$$|\Gamma_H|^2 = \frac{P_{refl}}{P_{inc}} = \frac{P_{rad}^-}{P_{inc}}, \quad (29)$$

the transmission coefficient in terms of power is

$$|T_H|^2 = \frac{P_{tr}}{P_{inc}} = \frac{P_{rad}^+}{P_{inc}}, \quad (30)$$

and the absorption coefficient in terms of power is

$$\alpha_{abs} = \frac{P_{abs}}{P_{inc}}. \quad (31)$$

5. Conclusions

The formulation presented herein allows for calculating reflection, transmission, and absorption coefficients for regular 3D arrays of magnetized or polarized ellipsoidal scatterers, whose sizes are small compared to the wavelength.

These scatterers are represented as three orthogonal elementary magnetic and/or three orthogonal elementary electric dipoles, abbreviated as EMR and EER. In particular, these scatterers can be small monocrystalline ferrite ellipsoids with pronounced uniaxial magnetic crystallographic anisotropy, such as hexagonal ferrites. The plane wave interacting with the array in this formulation can be of any angle of incidence and of any polarization.

The presented algorithm uses the small perturbation theory (when inhomogeneity is much smaller than the minimum considered wavelength) and the concepts of the coupling matrix and coupling coefficient between the dipole moments. In the 3D array, the phase shifts for the corresponding layers excitation are taken into account. The total field radiated by any EMR or EER includes both near-field and far-field terms. Though individual inhomogeneities are magnetically and electrically non-interacting in static fields, the assumption about electromagnetically non-interacting scatterers becomes invalid at higher frequencies, larger sizes of scatterers, and smaller distances between the individual scatterers. Magnitudes of a magnetic or electric moment of an individual scatterer are calculated through the coupling of the scatterer with both the incident wave and the fields produced by the other scatterers in the point where the given scatterer is placed. The particular case of an array of FRs is considered, but the formulation can be generalized for any system of small scatterers.

In future, the theory will be expanded to consideration of inhomogeneities with distributed equivalent electric and magnetic currents, rather than elementary magnetic and electric dipoles. This can be done using Galerkin integral equation method, approximating the corresponding surface currents by a number of physically reasonable basis functions.

The importance of this work is that it lays the theoretical basis for the analysis of electromagnetic wave scattering on any possible arrays of elements – not only magnetic, but also electric and coupled electric and magnetic dipoles inside the same inhomogeneity. Indeed, for modelling and design of frequency-selective surfaces and metamaterials (including those tunable) any type of elements in periodic structures can be used – made of magnetic materials, dielectrics, ferroelectrics, patches of different geometry on a homogeneous dielectric substrate, apertures of different geometry on a conducting sheets, simple-connected and multiple-connected structures that can be described both in terms of equivalent electric and magnetic currents.

Appendix A

External magnetic susceptibility tensors for a crystallographically anisotropic ferrite resonator

Consider the Cartesian coordinate system (123) related to the direction of the equilibrium magnetization $\vec{M}_0 \| \vec{3}^0$, as shown in Fig. 2. Another Cartesian coordinate system (xyz) is related to the direction of the bias external magnetic field $\vec{H}_0 \| \vec{z}^0$. Assume that the monocrystalline uniaxial FR with crystallographic axis c is magnetized up to saturation by the bias field $\vec{H}_0$. The conditions of $\vec{M}_0$ equilibrium result from the minimum of magnetic energy of the ferrite crystal. First, the vectors $\vec{M}_0, \vec{H}_0$, and $\vec{H}_A$ are co-planar, that is, $\varphi_H = \varphi_M = \varphi$. Second, the angles of the main vectors shown in Fig. 2 are related as $\sin\theta = H_A \sin(2\theta_0)/(2H_0)$, where $H_A = 2K_1/M_0$ is the magnetic crystallographic field determined through the first anisotropy constant K_1. The coordinate system (xyz) results from two rotations of the system (123): first rotation is on the angle $+\theta$ about the axis z, and second is on the angle φ around the axis y. The external susceptibility tensor relates external microwave (mm-wave) magnetic field and the corresponding microwave (mm-wave) magnetization as [3,4],

$$\vec{m} = \vec{\vec{\chi}}^{ext} \vec{h} \tag{A1}$$

of an ellipsoid made of a crystallographically isotropic ferrite in the system (123) is

$$\vec{\vec{\chi}}_m^{ext} = \begin{vmatrix} \chi_{11}^{ext} & j\kappa^{ext} & 0 \\ -j\kappa^{ext} & \chi_{22}^{ext} & 0 \\ 0 & 0 & 0 \end{vmatrix}, \tag{A2}$$

where for the non-zero Gilbert's loss parameter α_G, such that the FMR line width is

$$\Delta H_{0.5} = \frac{\alpha_G \omega_0}{\mu_0 \gamma} = \alpha_G H_0,$$

the components of the magnetic susceptibility tensor are calculated as

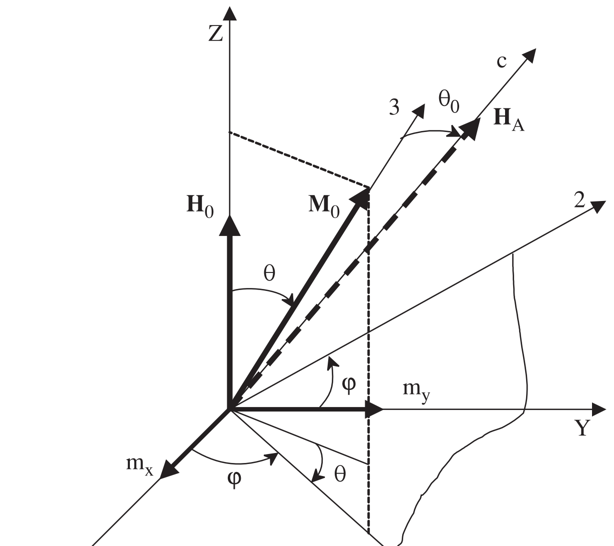

Fig. 2. Orientation of the main vectors describing magnetic susceptibility tensor of a uniaxial monocrystalline ferrite resonator.

$$
\begin{aligned}
\chi_{11,22}^{ext} &= \chi_{11,22}'^{ext} + j\chi_{11,22}''^{ext},\\
\chi_{11,22}''^{ext} &= \frac{\omega_{11,22}\omega_M(\omega_{res}^2 - (1+\alpha_G^2)\omega^2) + 2\omega_M\alpha_G^2\omega^2\omega_{res}}{\Delta},\\
\chi_{11,22}''^{ext} &= \frac{2\omega_{11,22}\omega_M\alpha_G\omega_{res} - \alpha_G\omega\omega_M(\omega_{res}^2 - (1+\alpha_G^2)\omega^2)}{\Delta},\\
\kappa_{11,22}^{ext} &= \kappa_{11,22}'^{ext} + j\kappa_{11,22}''^{ext},\\
\kappa'^{ext} &= \frac{\omega\omega_M}{\Delta}(\omega_{res}^2 - (1+\alpha_G^2)\omega^2 + 2\alpha_G\omega_{res}\omega_{12}),\\
\kappa''^{ext} &= \frac{2\omega^2\omega_M\omega_{res}\alpha_G - \omega_M\omega_{12}(\omega_{res}^2 - (1+\alpha_G^2)\omega^2)}{\Delta},\\
\Delta &= (\omega_{res}^2 - (1+\alpha_G^2)\omega^2)^2 + 4\alpha_G^2\omega^2\omega_{res}^2,\\
\omega_{res}^2 &= \omega_{11}\omega_{12} - \omega_{12}^2,\\
\omega_{11,12} &= \omega_0 + \omega_M N_{22,11};\ \omega_{12} = \omega_M N_{12}.
\end{aligned}
\tag{A3}
$$

In (A3), $\omega_0 = \mu_0\gamma H_0$ is the angular frequency corresponding to the bias magnetic field, $\omega_M = \mu_0\gamma M_0$ is the angular frequency associated with the magnetization of saturation, $\omega_A = \mu_0\gamma H_A$ is the angular frequency associated with the field of crystallographic anisotropy. The permeability of vacuum is $\mu_0 = 4\pi\times10^{-7}$ H/m and the gyromagnetic ratio is $\gamma = 1.76\times10^{11}$ C/kg.

The tensor of a crystallographically anisotropic uniaxial ferrite (without taking into account its shape) at an arbitrary orientation of its axis of magnetic anisotropy with respect to the bias magnetic field can be found using the equations in Refs. 6 and 7. In the initial Cartesian coordinate system (*x,y,z*), the susceptibility tensor is

$$
\ddot{\chi} = \begin{vmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz}\\ \chi_{yx} & \chi_{yy} & \chi_{yz}\\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{vmatrix}. \tag{A4}
$$

The components of this tensor can be obtained using the coordinate transformation matrix, where the direction of the equilibrium magnetization is determined by the angles φ and θ with respect to the axes *x* and *z*, correspondingly (see Fig. 2).

$$
\begin{aligned}
A_{zy} &= A_z A_y\\
&= \begin{bmatrix} \cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} \cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.
\end{aligned}
\tag{A5}
$$

$$
\ddot{\chi}_{xyz} = A_{zy}\ddot{\chi}_{123}A_{zy}^{-1}. \tag{A6}
$$

From Eq. (A5), the transformation matrix is

$$
A_{zy} = \begin{bmatrix} \cos\varphi\cos\theta & -\sin\varphi & \cos\varphi\sin\theta\\ \sin\varphi\cos\theta & \cos\varphi & \sin\varphi\sin\theta\\ -\sin\theta & 0 & \cos\theta \end{bmatrix}. \tag{A7}
$$

and the inverse matrix is

$$
A_{zy}^{-1} = \begin{bmatrix} \cos\varphi\cos\theta & \sin\varphi\cos\theta & \cos\varphi\sin\theta\\ -\sin\varphi & \cos\varphi & 0\\ \cos\varphi\sin\theta & \sin\varphi\sin\theta & \cos\theta \end{bmatrix}. \tag{A8}
$$

The components of the external tensor $\ddot{\chi}_{xyz}^{ext}$ can be found from Eqs. (A4)–(A8) as

$$
\begin{aligned}
\chi_{xx}^{ext} &= \chi_{11}^{ext}\sin^2\varphi + \chi_{22}^{ext}\cos^2\varphi\cos^2\theta,\\
\chi_{yy}^{ext} &= \chi_{11}^{ext}\cos^2\varphi + \chi_{22}^{ext}\sin^2\varphi\cos^2\theta,\\
\chi_{xy}^{ext} &= \frac{1}{2}(\chi_{22}^{ext}\cos^2\theta - \chi_{11}^{ext})\sin2\varphi + j\kappa^{ext}\cos\theta,\\
\chi_{yx}^{ext} &= \frac{1}{2}(\chi_{22}^{ext}\cos^2\theta - \chi_{11}^{ext})\sin2\varphi - j\kappa^{ext}\cos\theta,\\
\chi_{zx}^{ext} &= -\frac{1}{2}\chi_{11}^{ext}\sin2\theta\cos\varphi + j\kappa^{ext}\sin\theta\sin\varphi,\\
\chi_{xz}^{ext} &= -\frac{1}{2}\chi_{11}^{ext}\sin2\theta\cos\varphi - j\kappa^{ext}\sin\theta\sin\varphi,\\
\chi_{zy}^{ext} &= -\frac{1}{2}\chi_{11}^{ext}\sin2\theta\sin\varphi + j\kappa^{ext}\sin\theta\sin\varphi,\\
\chi_{yz}^{ext} &= -\frac{1}{2}\chi_{11}^{ext}\sin2\theta\sin\varphi - j\kappa^{ext}\sin\theta\sin\varphi,\\
\chi_{zz}^{ext} &= \chi_{11}\sin^2\varphi.
\end{aligned}
\tag{A9}
$$

Appendix B

Coupling between the FR and electromagnetic field

The reaction of a FR to the incident field can be taken into account using the self-matched field approach [3,4]. It is assumed that the magnetization of the FR placed in the point β is determined by the total magnetic field. This field which is the sum of the field of the incident wave, scattered by the other FRs, $\vec{h}_\beta = \vec{h}^{inc} + \vec{H}_{\Sigma/\beta}^{scat}$, and the field $\vec{H}_\beta^{scat}$ scattered (re-radiated) by the FR itself into the array space.

Since the FR is a small ellipsoid, its magnetization is supposed to be uniform, and it is related to the field through the FR's external susceptibility tensor,

$$\vec{m}_\beta = \vec{\vec{\chi}}_\beta^{ext}(\vec{h}_\beta + \vec{H}_\beta^{scat}). \tag{B1}$$

The field scattered by the β-th FR can be written as

$$\vec{H}_\beta^{scat} = -j\vec{\vec{w}}_\beta \vec{m}_\beta, \tag{B2}$$

where $\vec{\vec{w}}_\beta$ is the coupling tensor that depends on the interaction between the total array and an individual FR at the point β. For further convenience let us represent the magnetization vector of Eq. (B1) in terms of the equivalent susceptibility tensor and incident magnetic field,

$$\vec{m}_\beta = \vec{\vec{\chi}}_\beta^{eq}\vec{h}_\beta. \tag{B3}$$

Solving the system of Eqs. (B1) and (B2) for $\vec{m}_\beta$ in a matrix form as

$$\vec{m}_\beta = [\vec{\vec{I}} + j\vec{\vec{\chi}}_\beta^{ext}\vec{\vec{w}}_\beta]^{-1}\vec{\vec{\chi}}_\beta^{ext}\vec{h}_\beta, \tag{B4}$$

one can get the expression for the equivalent susceptibility tensor

$$\vec{\vec{\chi}}_\beta^{eq} = [\vec{\vec{I}} + j\vec{\vec{\chi}}_\beta^{ext}\vec{\vec{w}}_\beta]^{-1}\vec{\vec{\chi}}_\beta^{ext}, \tag{B5}$$

where the unit tensor is

$$\vec{\vec{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The scattered field $\vec{H}_\beta^{scat}$ can be represented as a sum of normalized propagating and evanescent (near field) waves – eigenmodes [9]. These are the eigenmodes of a semi-infinite 3D array of elements [2,13],

$$\vec{H}_\beta^{scat} = \sum_v C_v^{\pm}\vec{h}_v^{\pm}, \tag{B6}$$

where $v = (p,q)$ is the index of summation for the corresponding Floquet modes

$$\vec{h}_v = \vec{H}_v^{\max}\exp(-j\vec{k}_v^{\pm}\vec{r}_\beta)$$

with wave numbers

$$\vec{k}_v^{\pm} = \vec{k}_{p,q}^{\pm} = \pm k_v^x\vec{x}^0 + k_p^y\vec{y}^0 + k_q^z\vec{z}^0, \quad k_p^y = k_y + \frac{2\pi p}{D_y},$$

$$k_q^z = k_z + \frac{2\pi q}{D_z}, \quad k_{p,q}^x = \sqrt{k^2 - (k_p^y)^2 - (k_q^z)^2},$$

k is the wave number in the surrounding medium (in particular, free space), and $\vec{r}_\beta = x_\beta\vec{x}^0 + y_\beta\vec{y}^0 + z_\beta\vec{z}^0$ is the radius-vector to the point of observation β.

The coefficients of this representation are found using Vainshtein's approach valid for waveguides [9, Sections 76, 77],

$$C_v^{\pm} = -\frac{1}{N_v}\int_{V_\beta}\vec{p}_\beta^{\,m}\vec{h}_v^{\mp}dV, \tag{B7}$$

where N_v are the norms of the eigenmodes. Vainshtein's approach can be extended for more complex structures rather than waveguides, if the full set of eigenmodes is known [9, Section 78]. Herein, the full set of Floquet harmonics for a periodic array is used, and $\vec{p}^{\,m}$ is the magnetic dipole moment (or magnetic current) determined by Eqs. (1) and (2),

$$\vec{p}_\beta^{\,m} = j\omega\mu_0\vec{m}_\beta\delta(\vec{r} - \vec{r}_\beta)V_\beta e^{-j\vec{k}\vec{r}_\beta}. \tag{B8}$$

A norm of a Floquet mode is determined through the condition of orthogonality of the full set of modes, and is proportional to the complex power of the mode passing through some unit cross-section S [9, Section 75],

$$N_v = -4P = -2\int_S e_v \times h_v^* ds, \tag{B9}$$

where e_n, h_n^* are the complex amplitudes of the microwave electric and conjugated magnetic field of the v^{th} Floquet mode, respectively. For the local scattered waves by the FR at the impact of a plane wave, the individual norms should be calculated as in Eq. (B9), while the fields can be found using, for example, the rigorous approach described in Ref. 13, or the approach in Ref. 15 for an elementary radiator (a magnetic dipole or an electric dipole) in the vicinity of a planar dielectric layer (or a multilayered structure).

Then, the coefficients of Eq. (B7) can be calculated as

$$\begin{aligned} C_v^{\pm} &= -\frac{1}{N_v}\int_{V_\beta} V_\beta j\omega\mu_0\vec{m}_\beta\delta(\vec{r} - \vec{r}_\beta)\vec{h}_v^{\mp}e^{-j\vec{k}\vec{r}_\beta}dV \\ &= \frac{-j\omega\mu_0 V_\beta \exp\left(\pm j\left(k_{pq}^x x + \dfrac{2\pi p}{D_y} + \dfrac{2\pi q}{D_z}\right)\right)}{N_v} \times \\ &\quad (m_x H_{vx}^{\max} + m_y H_{vy}^{\max} + m_z H_{vz}^{\max})_{\vec{r}_\beta}. \end{aligned} \tag{B10}$$

The scattered fields in the positive and negative directions, according to Eq. (B6), are

$$\begin{aligned} \vec{H}_\beta^{scat} &= -j\omega\mu_0 V_\beta e^{-j\vec{k}\vec{r}_\beta} \cdot \\ &\sum_v \frac{\exp\left(\pm j\left(k_{pq}^x x + \dfrac{2\pi p}{D_y} + \dfrac{2\pi q}{D_z}\right)\right)}{N_v} \times \\ &(m_x H_{vx}^{\max} + m_y H_{vy}^{\max} + m_z H_{vz}^{\max})_{\vec{r}_\beta}. \end{aligned} \tag{B11}$$

The total coupling tensor $\vec{\vec{w}}_\beta$ is obtained from Eqs. (B3) and (B11),

$$\vec{\vec{w}}_{\beta} = \omega\mu_0 V_{\beta} e^{-j\vec{k}\vec{r}_{\beta}} \begin{bmatrix} \sum_{v} \frac{(H_{vx}^{\max})^2}{N_v} & \sum_{v} \frac{H_{vx}^{\max} H_{vy}^{\max}}{N_v} & \sum_{v} \frac{H_{vx}^{\max} H_{vz}^{\max}}{N_v} \\ \sum_{v} \frac{H_{vx}^{\max} H_{vy}^{\max}}{N_v} & \sum_{v} \frac{(H_{vy}^{\max})^2}{N_v} & \sum_{v} \frac{H_{vy}^{\max} H_{vz}^{\max}}{N_v} \\ \sum_{v} \frac{H_{vx}^{\max} H_{vz}^{\max}}{N_v} & \sum_{v} \frac{H_{vy}^{\max} H_{vz}^{\max}}{N_v} & \sum_{v} \frac{(H_{vz}^{\max})^2}{N_v} \end{bmatrix}_{\vec{r}_{\beta}} . \tag{B12}$$

As it can be seen from Eq. (B12), the coupling tensor is determined by the volume of the FR, Floquet mode structure, frequency of the electromagnetic signal, and a point in the array, where the ferrite resonator is placed.

Following from Eq. (B5), the equivalent magnetic susceptibility tensor for the β-th FR in the array is related to its external tensor through the coupling coefficient $\eta_{c\beta}$,

$$\chi_{\beta ij}^{eq} = \frac{\chi_{\beta ij}^{ext}}{1+\eta_{c\beta}}, \tag{B13}$$

where the coupling coefficient $\eta_{c\beta}$ is calculated through the coupling tensor $\vec{\vec{w}}_{\beta}$ as in Eq. (20),

$$\eta_{c\beta} = \det\{\vec{\vec{I}} + j\vec{\vec{\chi}}_{\beta}^{ext} \vec{\vec{w}}_{\beta}\} - 1. \tag{B14}$$

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