Approximate analysis of nonlinear operation of Fabry-Perot laser with saturable absorber

Part I: Output power optimization

P. SZCZEPAŃSKI*), L. WOSIŃSKA**) and W. WOLIŃSKI*)

*) Institute of Microelectronics and Optoelectronics, Warsaw University of Technology
00-662 Warszawa, ul. Koszykowa 75, Poland

**) Department of Telecommunication and Computer Systems, The Royal Institute of Technology
Box 700043, S-100 44 Stockholm, Sweden

1. Introduction

Optical bistability has received much attention from researchers as well as theorists during the past decade. Bistable optical devices may play an important role in such fields as optical computing in addition to optical signal processing and photonic switching in optical communication systems. Bistable optical devices may potentially be used as optical transistors, memory elements, and logic elements.

Most bistable optical devices built until today require both an optical resonator and a nonlinear medium to operate. Passive devices have been demonstrated in both nonlinear dispersive [1, 2] and absorptive media [3]. The first active devices, i. e. laser resonators containing a saturable absorber, were demonstrated over 20 years ago [4, 5]. Experimental results from experiments on instabilities in lasers with saturable absorbers have stimulated continuing theoretical research [6–8] aimed at describing time-dependent behavior observed in this kind of laser. Steady-state analysis of the performance of lasers with the saturable absorber developed in Ref. [9–12] have been limited to the mean field approximation. This is due to the fact that, in this case the laser field is assumed to vary slowly over the length of the resonator. A laser with a uniform intensity inside the cavity is extremely appealing because of its analytical simplicity.

However, in a two-mirror laser with a saturable absorber, the uniform intensity model could give an incorrect estimate of the laser intensity and the output power characteristics. Additionally, in inherently high gain systems (for example semiconductor lasers) operating as a bistable device, the single-pass loss in the nonlinear absorber may become appreciable, causing a considerable change in the field intensity within the laser cavity.

Models using laser containing a saturable absorber, and also take into account the axial field dependence have been developed in Ref. [13]. Results were obtained by numerical integration of the standard propagation equations for laser intensity in an amplifying and absorbing medium, taking into account boundary conditions at the two resonant end mirrors. This approach, although providing specific solutions, is hardly useful when designing real bistable structures, due to the large number of characteristic parameters involved in the description.

In this paper we will present an approximate analysis of a two-mirror laser with a saturable absorber inside the cavity. Our approach is based on the energy theorem and threshold field approximation, developed earlier for various structures of interest [14–16]. It provides results agreement with exact solutions for Fabry-Perot lasers [14] as well as for distributed feedback lasers [15, 16] for the values of the system parameters, which are likely to be of practical interest. In particular, less then 20% error was found for two-mirror lasers [14]. Our paper is divided into two parts. In the first part we derive an approximate expression showing the relation between small signal (unsaturable) gain output power normalized to saturation power in the active medium and system parameters. Saturable losses in the structure are taken into account. Using this expression, we derive the laser characteristics. The derived characteristics reveal the optimal output coupling reflectivity which in turn provides maximum output power for a given pumping rate for the active medium as a function of the system parameters such as linear loss in the structure, small signal loss coefficient (describing the saturable losses at zero light intensity level), saturation power of the absorptive medium as well as the length of the laser.

In the second part of this paper we discuss the conditions for bistable operation of the laser as a function of the system parameters.

Part I. of this paper is organized as follows. In the next Section we derive the approximate expression for small signal gain with the help of the energy theorem and threshold field approximation. The expression takes into account nonlinear gain as well as loss saturation in the laser medium. In Section 3, the laser characteristics are presented, revealing the optimal coupling strength which provides the maximal power efficiency of the structure. Conclusions are presented in Section 4.

2. Energy theory

We begin our analysis with the basic Fabry-Perot laser equation. The equation for field amplitudes inside the cavity taking into account nonlinear gain saturation as well as nonlinear loss saturation can be written as follows

$$\frac{dR}{dz}+(-\alpha^{gain}+\alpha^{loss}-\alpha_L^{loss}+i\delta)\,R = 0, \tag{1}$$

$$-\frac{dS}{dz}+(-\alpha^{gain}+\alpha^{loss}+\alpha_L^{loss}+i\delta)\,S = 0, \tag{2}$$

where R and S are the complex amplitudes of two counterrunning waves of the laser mode, δ is the frequency parameter describing the frequency shift of the mode from the passive cavity resonance and α_L^{loss} denotes the coefficient of the nonsaturable distributed, (linear) losses in the structure. For homogeneous broadening, with spatial hole burning effects neglected, the saturated gain α^{gain} in the medium can be related to the small signal gain α_0^{gain} for central tuning by the following equation

$$\alpha^{gain} = \frac{\alpha_0^{gain}\mathscr{F}_g(z)}{1+(|R|^2+|S|^2)/P_s^{gain}}, \tag{3}$$

where P_s^{gain} is the saturation power of the active medium and $\mathscr{F}_g(z)$ is the normalized function describing spatial distribution of the small signal gain in the active medium (which depends on the pumping).

Similarly, we can express the saturable losses α^{loss} in the structure by the small signal loss coefficient α_0^{loss} in the following equation

$$\alpha^{loss} = \frac{\alpha_0^{loss}\mathscr{F}_l(z)}{1+(|R|^2+|S|^2)/P_s^{loss}} \tag{4}$$

where P_s^{loss} is the saturation power of the nonlinear absorber and $\mathscr{F}_l(z)$ describes the spatial distribution of the small signal loss coefficient in the absorptive medium (for example, this function can describe the spatial distribution of absorptive centers in the medium).

We normalize R and S in so that the expression $|R(z)|^2+|S(z)|^2$ describes the total power in the structure. Now, multiplying equation (1) by R^* (complex conjugate of R) and Eq. (2) by S^* and adding these two equations and their conjugates, we obtain

$$\frac{d}{dz}(|R|^2-|S|^2) = 2\left(\frac{\alpha_0^{gain}\mathscr{F}_g(z)}{1+(|R|^2+|S|^2)/P_s^{gain}}+\right.$$
$$\left.-\frac{\alpha_0^{loss}\mathscr{F}_l(z)}{1+(|R|^2+|S|^2)/P_s^{loss}}-\alpha_L^{loss}\right)(|R|^2+|S|^2). \tag{5}$$

The boundary conditions for our structure can be described by the following equations

$$|R(L)|^2r_1^2 = |S(L)|^2, \quad |S(0)|^2r_2^2 = |R(0)|^2, \tag{6}$$

$$(1-r_1^2)|R(L)|^2 = P_{out}^R, \quad (1-r_2^2)|S(0)|^2 = P_{out}^S, \tag{7}$$

where r_1 and r_2 are the real amplitude reflectivities of the end mirrors; $P_{out}^R+P_{out}^S=P^{out}$ describes the total power escaping from the structure; and L is the length of the laser cavity. Integration of Eq.(5) from $z=0$ to $z=L$, while taking into account boundary conditions, described by Eqs. (6) and (7), gives,

$$(1-r_1^2)|R(L)|^2+(1-r_2^2)|S(0)|^2 = 2\int_0^L dz\,\times$$
$$\times\left(\frac{\alpha_0^{gain}\mathscr{F}_g(z)}{1+(|R|^2+|S|^2)/P_s^{gain}}-\frac{\alpha_0^{loss}\mathscr{F}_l(z)}{1+(|R|^2+|S|^2)/P_s^{loss}}-\alpha_L^{loss}\right)\times$$
$$\times(|R|^2+|S|^2). \tag{8}$$

This relation is exact and states that power escaping from the ends of the structure is equal to the net power generated inside the structure. We will use relation as a starting point for our approximate analysis.

For our purposes, we will approximate the field distribution appearing in the energy theorem, Eq. (8), including nonlinearities, by the one existing in the linear structure. In other words, we will satisfy all linear equations (valid at the threshold laser operation), with $\alpha^{gain}=\alpha_0^{gain}$ and $\alpha^{loss}=\alpha_0^{loss}$) for the following boundary conditions

$$|R(L)|r_1 = |S(L)| \quad \text{and} \quad |S(0)|r_2 = |R(0)|$$

It is worth noting, that the threshold operation in the case of the bistable device is equivalent to the switch-up point (see Part II).

Thus we assume that the field amplitudes R and S, appearing in Eq. (8), are proportional to the threshold field distribution and have unknown amplitude A. This gives us

$$|R(z)| = |A|\exp(\gamma z) \quad \text{and} \quad |S(z)| = \frac{|A|}{r_2}\exp(-\gamma z), \tag{9}$$

$$\text{where} \quad \gamma = (1/2L)\ln[1/(r_1r_2)] = \alpha_0^{gain}-\alpha_0^{loss}-\alpha_L^{loss} \tag{10}$$

It should be noted that this approach has been verified earlier for Fabry-Perot [14] and distributed feedback lasers [15, 16] with linear losses.

Taking into account boundary conditions, Eq. (7), and the field distribution, Eq. (9), we can relate the amplitude A to the output power in the following way

$$|A|^2 = r_2\frac{P^{out}}{\dfrac{(1-r_1^2)}{r_1}+\dfrac{(1-r_2^2)}{r_2}}. \tag{11}$$

Combining the approximate field distribution for R and S, Eq. (9), and the expression for field amplitude, Eq. (11), with the energy theorem, Eq. (8), we obtain

$$2\alpha_0^{gain} = \left\{ 1/r_2 \left[\frac{(1-r_1^2)}{r_1} + \frac{(1-r_2^2)}{r_2} \right] + \right.$$

$$+ 2 \int_0^L dz\, \alpha_0^{loss}(z)\, [|f_R(z)|^2 - |f_S(z)|^2] +$$

$$\left. - 2 \int_0^L dz \frac{\alpha_0^{loss} \mathscr{F}_l(z)}{1 + \frac{P^{out}}{P_s^{gain}} \mathscr{N} \beta [|f_R(z)|^2 + |f_S(z)|^2]} \right\} \times$$

$$\times \left\{ \int_0^L dz \frac{\mathscr{F}_g(z)}{1 + \frac{P^{out}}{P_s^{gain}} \mathscr{N} [|f_R(z)|^2 + |f_S(z)|^2]} \right\}^{-1}, \qquad (12)$$

where

$$f_R(z) = \exp(\gamma z) \quad \text{and} \quad f_S(z) = \frac{1}{r_2} \exp(-\gamma z),$$

parameter β describes the ratio of the amplifier to absorber saturation intensity, $\beta = P_s^{gain}/P_s^{loss}$, and the normalization factor is defined as

$$\mathscr{N} = \frac{r_2}{\frac{(1-r_1^2)}{r_1} + \frac{(1-r_2^2)}{r_2}}.$$

For the normalized small signal gain coefficient $\alpha_0^{gain} L$, the normalized distributed linear coefficient $\alpha_L^{loss} L$, the normalized small signal loss coefficient $\alpha_0^{loss} L$, and the normalized propagation constant $\gamma L = \Gamma$ we can rewrite Eq. (12) as:

$$2\alpha_0^{gain} L = \left\{ 1/r_2 \left[\frac{(1-r_1^2)}{r_1} + \frac{(1-r_2^2)}{r_2} \right] + \right.$$

$$+ 2\alpha_0^{loss} L \int_0^1 d\xi\, [|f_R(\xi)|^2 + |f_S(\xi)|^2] +$$

$$\left. - 2\alpha_0^{loss} L \int_0^1 d\xi \frac{\mathscr{F}_l(\xi L)}{1 + \frac{P^{out}}{P_s^{gain}} \mathscr{N} \beta [|f_R(\xi)|^2 + |f_S(\xi)|^2]} \right\} \times$$

$$\times \left\{ \int_0^1 d\xi \frac{\mathscr{F}_g(\xi L)}{1 + \frac{P^{out}}{P_s^{gain}} \mathscr{N} [|f_R(\xi)|^2 + |f_S(\xi)|^2]} \right\}^{-1}, \qquad (13)$$

where

$$f_R(\xi) = \exp(\Gamma \xi) \quad \text{and} \quad f_S(\Gamma \xi) = \frac{1}{r_2} \exp(-\Gamma \xi).$$

Equation (13) is an approximate expression, describing the normalized small signal gain coefficient $2\alpha_0^{gain} L$ as of output power normalized to the saturation power of the active medium, P^{out}/P_s^{gain} and system parameters. It takes into account gain saturation as well as saturation of the loss coefficient in the nonlinear absorptive medium. In the next section we use this expression to obtain output power characteristics.

3. Numerical results

In this section we describe the results of a numerical integration of equation (13). The results are obtained with the assumption that we have uniform distribution of the small signal gain and the small signal loss coefficients. Similarly the constant linear distributed loss coefficient is assumed. Moreover, we put $r_2 = 1$.

In Fig. 1a., the normalized small signal gain coefficient $2\alpha_0^{gain}$ is plotted against the the output mirror reflectivity r_1 with the normalized distributed linear loss coefficient $\alpha_L^{loss} L$ as a parameter. The normalized output power is $P^{out}/P_s^{gain} = 1$, the normalized small signal loss coefficient is $\alpha_0^{loss} L = 1$ and $\beta = 2$. It is worth noting that $2\alpha_0^{gain} L$ plots exhibit a minimum within the presented range of the output mirror reflectivity r_1. Thus, for given output power level (keeping other parameters of the structure constant) an optimum coupling strength exists. This results in a minimum small-signal gain required to maintain the output-power level. Correspondingly, there is a maximum output-power level for a given pumping rate of the active medium. Thus, for optimum coupling strength the laser structure has a corresponding maximum power efficiency.

It should also be noted that in general, this condition (i. e. maximum power efficiency) can help to increase the operating speed of the bistable laser device. In this case, however, the laser structure operates at the given output power level with the smallest pumping rate of the active medium. This condition is important when designing a bistable laser device for high-speed operation.

Moreover, the optimum coupling strength (i. e. the optimum output mirror reflectivity) depends on the linear loss level. With the increasing linear losses the minimum becomes broader and shifts towards smaller values of output mirror reflectivity. Thus, for higher linear losses the power efficiency becomes less sensitive to the value of the output mirror reflectivity. On the other hand, however, the small signal gain required to maintain given output power level increases and the laser structure operates with a higher pumping rate of the active medium.

The plots of Fig. 1b show the dependence of normalized small signal gain $2\alpha_0^{gain} L$ as a function of output mirror reflectivity, with normalized small signal losses $\alpha_0^{loss} L$ as a parameter. The other parameters of the structure are: $P^{out}/P_s^{gain} = 1$, $\beta = 2$, and $\alpha_L^{loss} L = 0.01$. Similar to the previous case an optimum value of the output mirror reflectivity exists. Note that the characteristics are less sensitive to the changes of the small signal losses than to the changes of the linear losses (Fig. 1a). This fact may be convenient when designing of a real laser acting as a bistable device, since the changes of the small signal loss coefficient (as occurs during modelling of the hystheresis loop of the device), do not significantly affect its power efficiency.

In the next figure, Fig. 1c, the normalized small signal gain coefficient $2\alpha_0^{gain} L$ is plotted as a function of the output mirror reflectivity r_1, with the normalized output power P^{out}/P_s^{gain} as a parameter. One can clearly see that the minimum depends on the output power level. With increasing output power the minimum becomes broader and the optimal output mirror reflectivity is shifted towards the lower values. Moreover, we observe an interesting behavior in the characteristics. Note that for small coupling strength (i.e. for small values of the output mirror reflectivity) there is an inversion of characteristics. When output power is increased, the small signal gain required decreases. This is a manifestation of nonlinear losses decreasing faster than the gain available in the structure ($\beta > 1$).

The inversion of the characteristics disappears at higher output power levels, since in this region the gain available in the structure saturates significantly and a higher level of small signal gain is required to provide a given output power level. Thus in this region of operation the laser structure exhibits bistable behavior. Inversion of the characteristics disappers also at a given power level with increasing output mirror reflectivity. In this case, the field intensity inside the cavity increases simultaneously and saturates gain as well as losses in the structure. As a result, the region of bistability is shifted toward the smaller values of output power.

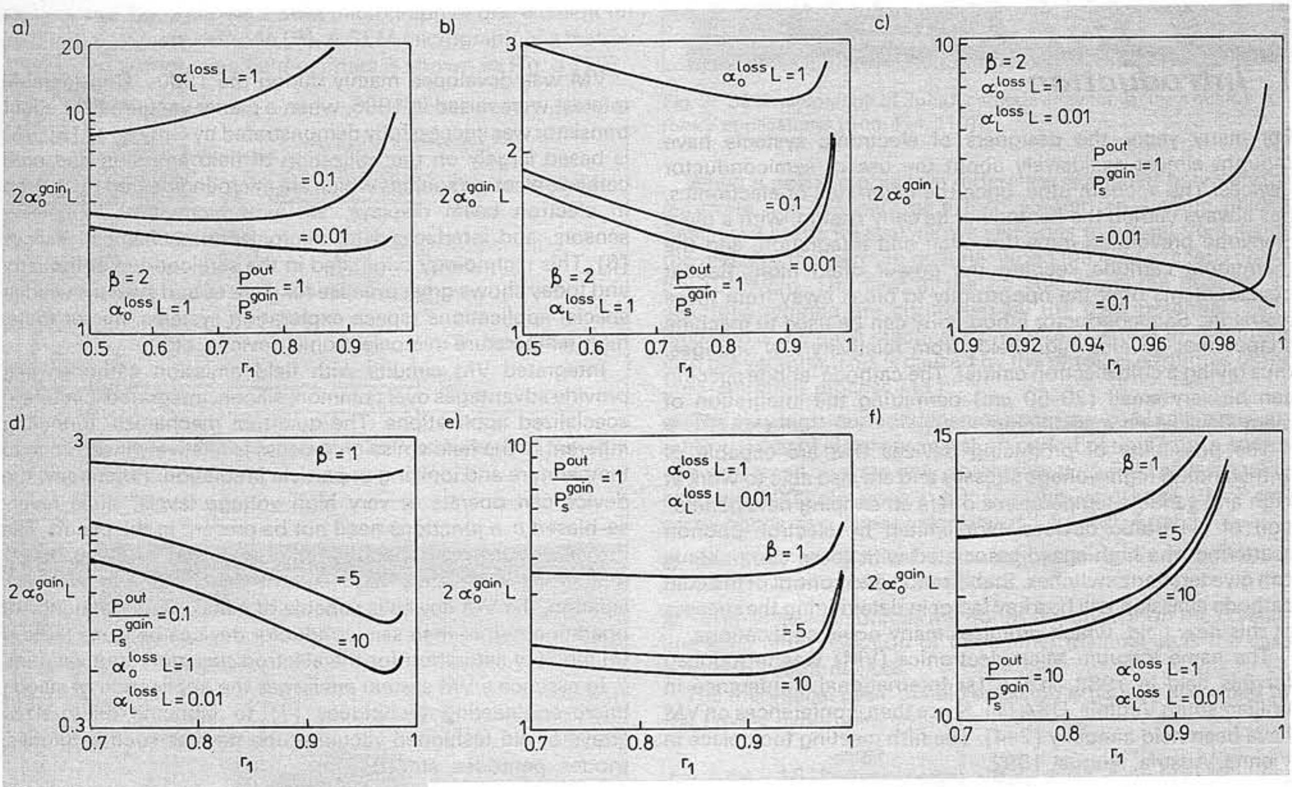
Fig. 1d shows the normalized small signal gain $2\alpha_0^{gain} L$ as a function of output mirror reflectivity r_1, with the ratio of saturation power in the active medium to the saturation power in the absorptive medium, β, as a parameter. The normalized output power is $P^{out}/P_s^{gain} = 0.01$, the normalized small signal loss coefficient is $\alpha_0^{loss} L = 1$, and the normalized linear loss coefficient is $\alpha_L^{loss} L = 0.01$. With the increasing value of parameter β the small signal gain required to maintain a given output power level decreases. This behavior is obvious since the level of losses in the structure decreases simultaneously due to the increase in loss saturation. It is worth noting that the output mirror reflectivity is shifted toward greater values when β increases, and that it tends toward the value limited by linear losses. This is caused because the loss level in the structure decreases simultaneously due to the loss saturation effect.

Moreover, at higher output power levels, Figs. 1e and f, the characteristics become less sensitive to changes of parameter β since the increasing field intensity inside the laser cavity saturates the nonlinear loss more effectively.

4. Conclusions

In this paper, we have presented an approximate method to analyse the operation of a laser with a saturable absorber. Taking onto account axial field dependence in the cavity, we extend the analysis method of the nonlinear operation of such a laser device beyond the mean field approximation.

Using the energy theorem and threshold field approximation, an approximate formula for the normalized signal gain coefficient $2\alpha_0^{gain} L$ is obtained as a function of the system parameters. Laser characteristics revealing the optimum output mirror reflectivity providing maximum power efficiency of structure have been obtained. It has been shown that changes in the



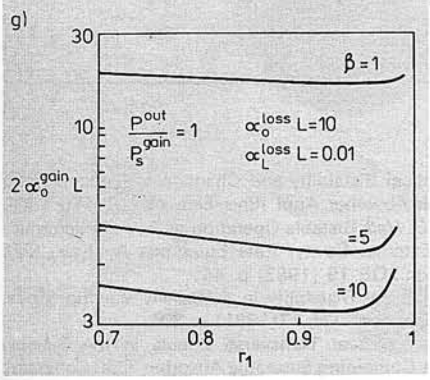


Fig. 1. Normalized small signal gain versus output mirror reflectivity for different parameters

In the next figure, Fig. 1g, similar characteristics are shown as in Figs. 1d–f, however, in this case the value for the normalized small signal loss coefficient $\alpha_0^{loss} L$ is increased and is now $\alpha_0^{loss} L = 10$. As in the case of increased output power level, the increase in small signal loss coefficient in the laser structure makes the optimum value of the output mirror reflectivity less sensitive to the changes of parameter β.

Thus we can say that for higher output power levels as well as for higher levels of small signal loss coefficient, changes of parameter β during modelling, the hysteresis of the bistable laser device, do not significantly affect the power efficiency of the structure.

small signal loss coefficient (characterizing the absorptive nonlinear medium) do not significantly affect the power efficiency of the structure, i. e. they do not cause important changes in the optimum value of the output mirror reflectivity.

Similarly, for higher output levels and higher small signal losses in the structure, the optimum output mirror reflectivity does not depend on the ratio of saturation power of the active medium to the saturation power of the absorptive medium, i. e. parameter β. This fact can be helpful, for an example, when modelling the hystheresis loop of a bistable device without significant changes in the power efficiency of the structure. On the other hand, designing a bistable laser structure with the optimum output mirror reflectivity (i. e. at the optimum power efficiency) can help in obtaining higher operation speed since the laser structure works at the lower pumping rate of the active medium.

We believe that the presented technique facilitates the design of bistable laser devices and the calculation of the optimum output mirror reflectivity providing maximum power efficiency in the structure.

Acknowledgments

The authors wish to express their thanks to Andres Hedin for his support in carrying out this work.

▶

References

1. *H.M. Gibbs, S.S. Tarng, J.L. Jewell, D.A. Weinberg, K. Tai, A.C. Gossard, S.L. McCall, A. Passner* and *W. Wiegmann*: Room-Temperature Excitonic Optical Bistability in a GaAs-GaAlAs Superlattice Etalon. Appl. Phys. Lett., **41** (1982) p. 221.
2. *A.B. Miller, D.S. Chemla, D.J. Eilenberger, P.W.Smith, A.C. Gossard* and *W.T. Tsang*: Large Room-Temperature Optical Nonlinearity in GaAs/Ga_{1-x}AlAs Multiple Quantum Well Structures. Appl. Phys. Lett., **41** (1982) p. 769.
3. *A.T. Rosenberger, L.A. Orozco* and *H.J. Kimble*: Observation of Absorptive Bistability with Two-Level Atoms in a Ring Cavity. Phys. Rev.,**A28** (1983) p. 2569.
4. *H.J. Geritsen*: Operation of a Memory Element Based on the Maser Principle. Proc. IEEE, **51** (1963) p. 934.
5. *N.I. Nathan, J.C. Marinace, R.F. Rutz, A.E. Michel* and *G.J. Lasher*: GaAs Injection Laser with Novel Mode Control and Switching Properties. J.Appl. Phys., **36** (1965) p. 473.
6. *E. Arimondo, F. Casagrande, L.A. Lugiato* and *P. Gorieux*: Repetive Passive Q-Switching and Bistability in Lasers with Saturable Absorbers. Appl. Phys., **B30** (1983) p. 57.
7. *P. Mandel* and *T. Erneux*: Stationary Harmonic and Pulsed Operation of Optically Bistable Laser with Saturable Absorber I. Phys. Rev., **A30** (1984) p. 893.
8. *H. Kawaguchi*: Optical Bistability and Chaos in a Semiconductor Laser with Saturable Absorber. Appl. Phys. Lett., **45** (1984) p. 1264.
9. *T.G. Dziura* and *D.G. Hall*: Bistable Operation of Two Semiconductor Lasers in an External Cavity: Rate Equations Analysis. IEEE J. Quantum Electron., **QE-19** (1983) p. 441.
10. *P.D. Durmmond*: Optical Bistability in a Radially Varying Mode. IEEE J. Quantum Electron., **QE-17**(1981) p. 301.
11. *D.G. Hall* and *T.G. Dziura*: Transverse Effects in the Bistable Operation of Lasers Containing Saturable Absorber. Opt. Commun., **49** (1984) p. 146.
12. *T.G. Dziura* and *D.G. Hall*: Semiclassical Theory of Bistable Semiconductor Lasers Including Radial Mode Variation. Phys. Rev., **A31** (1985) p. 1551.
13. *T.G. Dziura*: Beyond Mean Field and Plane Wave Theories of Bistable Semiconductor Lasers. IEEE J. Quantum Electron., **QE-22** (1986) p. 651.
14. *A. Kujawski* and *P. Szczepański*: A simple Model of Gain Saturation in Homogeneously CW Lasers with Distributed Losses. J. Mod. Opt., in press.
15. *P. Szczepański*: Approximate Analysis of Nonlinear Operation of a Distributed Feedback Laser with Parasitic Losses. J. Appl. Phys., **63** (1988) p. 4854.
16. *P. Szczepański, D. Sikorski* and *W. Woliński*: Nonlinear Operation of a Planar Distributed Feedback Laser: Energy Approach. IEEE J. Quantum Electron., **QE-25** (1988) p. 871.