Application of piezoelectric transducers for parametric sources

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1. Introduction

Piezoelectric transducers are the most common sources of volume acoustic waves in underwater acoustics. Also broadband receivers called hypodrophones are made with piezoelectric ceramics used as sensors. Most of the present hydroacoustic devices are piezolectric ceramic used as sensors. Most of the present hydroacoustic devices are designed on the basis of linear acoustic theorem. But more and more frequently they are complemented with the new class of devices in which phenomenon of nonlinear interaction of acoustic waves in fluid is utilized.

The most effective way of taking use of the phenomenon of nonlinear interaction of acoustic waves in the application of the parametric arrays as sources of acoustic waves. In such sources the nonlinear interaction of two waves of high frequency and high intensity, called the scattering of sound by sound, is used to produce low frequency wave equal to the difference of the frequencies of the primary waves.

The function of the piezoelectric transducers in parametric sources is different then that in the classical ones. They constitute the primary wave source of high frequency and high intensity, and not – the wave directly emitted to the medium.

Therefore it is desirable that the ceramics used for the production of sources of primary waves should have the following characteristics:

1. A strong as possible piezoelectric effect, anabling to cope with the necessity of transforming high power electrical signals into the acoustic ones with the use of small dimension transducer,
2. Small mechanical and dielectric losses, which feature is necessary because of temperature rising of the transducer and lowering of coefficiens of electromechanical and mechanoacoustic efficiency,
3. High stability of material constant, both in time and at work using signals of high amplitude and power.

In practice it is very difficult to comply with all these demands. The basic criterium for choosing piezoceramic material for the primary wave source in the parametric array should be the ability to obtain maximum acoustical power and the efficiency in transimitting electric energy into mechanical one.

2. Characteristics of a parametric acoustic source

The principle of functioning of a parametric source is illustrated in Fig. 1. The emitting parametric array is composed of a piezoceramic transducer emitting primary waves of high intensity and the part of the medium in which the nonlinear interaction of waves occurs, causing the creation of usable wave propagating beyond the area of interaction. Both of elements, i.e., the transducer emitting energy and the volume of the medium in which transforming of the energy to the secondary wave takes place – that is the process of actual forming to the directivity pattern of radiation – both of these elements together constitute the parametric source of acoustic waves.

Usually, the primary wave emitted into water consist of two components of different closely spaced frequencies f_1 and f_2, and the secondary wave is produced in the frequency equal to the difference of their frequencies.

The primary beam is assumed to be plane up to the distance of R_0 (the Rayleigh length) and spherically spreading beyond that range. There are four distinct operating modes for parametric sources. The effective length can be determined either by small-signal absorption or nonlinear absorption in either the near field or the far field of the primary beam.

Parametric sources applied in hydroacoustic are usually of the type limiting the effective length of interaction area by small-signal absorption in the far field.

The theoretical method in analyzing the interaction between finite amplitude waves of closely spaced frequencies is based on the solution of the nonlinear equations of acoustics. The equation is obtained from the general equations of fluid mechanism which are the equations of continuity, motion and state. Assuming small relative changes of density and pressure, the obtained nonlinear equation of acoustics takes the following form [20]:

$$\Delta p' - \frac{1}{c_o^2}\frac{\delta^2 p'}{\delta t^2} + \frac{b}{c_o^2\rho_0}\frac{\delta}{\delta t}\Delta p' = -Q \tag{1}$$

where:

$$Q = \frac{1}{c_o^4\rho_0}\left(\frac{\delta p'}{\delta t}\right)^2 + \frac{B}{2Ac_o^4\rho_0}\frac{\delta^2 p'}{\delta t^2} + \frac{\rho_0}{2}\Delta v^2 + \rho_0\vec{v}\,\Delta\vec{v} \tag{2}$$

and $p = p - \rho_0$ is acoustic pressure, c_0 is speed of sound wave, τ_0 is medium density at rest, b is attenaution factor, t is time, v is vibration velocity, B/A is nonlinearity parameter.

The solution for the equations (1) and (2) has still not been found. There are known simplified form of the equation of nonlinear acoustics. The Khokhlov-Zabolotska-Kuznecov equation was obtained on the basis of the quasi-optical assumption. In Lagrangian coordinates it is given in the following form [2]:

$$\frac{\delta}{\delta\tau}\left[\frac{\delta p'}{\delta z} - \frac{B/\mathrm{A}+2}{2c_0^3\rho_0}p'\frac{\delta p'}{\delta\tau} - \frac{b}{2c_0^3\rho_0}\frac{\delta^2 p'}{\delta\tau^2}\right] = \frac{c_0}{2}\left(\frac{\delta^2 \mathrm{p}'}{\delta \mathrm{x}^2} + \frac{\delta^2 \mathrm{p}'}{\delta \mathrm{y}^2}\right) \tag{3}$$

where: $\tau = t - z/c_0$ is time in the coordinate system fixed in the zero phase of the propagating wave, z is wave propagation direction and x, y are axes perpendicular to z.

This equation corresponds to the Burgers equation for onedimensional case. The generalized formula is following [4]:

$$\frac{\delta p'}{\delta z} + \frac{n}{2z}p' - \frac{b}{2c_0^3\rho_0}\frac{\delta^2 p'}{\delta\tau^2} - \frac{B/A+2}{2c_0^3\rho_0}p'\frac{\delta p'}{\delta\tau} = 0 \tag{4}$$

where $n=0$ for plane waves, $n=1$ for cylindrical and $n=2$ for spherical ones. It is a nonlinear equation that can be solved analytically. Its solution is very useful in analyzing the nonlinear effects occuring in plane, cylindrical or spherical.

The primary acoustic beam consisting of two discrete frequency components is usually produced by a piston transducer.

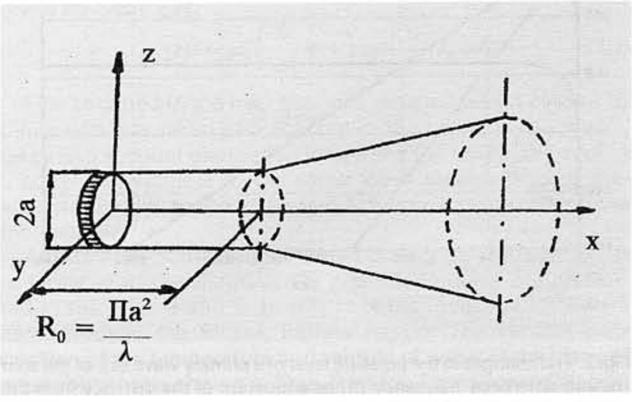

Fig. 1. The principle of functioning of a parametric source

The near field of the primary beam is approximated by a collimated attenuated plane wave of which the cross-sectional area is the same as that of the piston. Nonlinear interaction between primary waves causes in the medium the appearance of secondary waves of frequences $2f_1$, $2f_2$, f_1+f_2, f_1--f_2. Solving Burgers equation (4) for primary wave defined as:

$$p'(z=0,\ \tau)=p_{01}\cos(2\pi f_1\tau)+p_{02}\cos(2\pi f_2\tau),$$
$$F=f_1-f_2\ll f_1, f_2 \tag{5}$$

by means of the perturbation method we can obtain expressions pressure amplitudes of second harmonics of the primary wave components, and of the sum- and difference-frequency waves. The pressure amplitude of difference-frequency wave (the secondary signal of interest) is given as follows [20]:

$$p_F=\frac{p_{01}p_{02}\Omega}{b(\omega_2^1=\omega_2^2-\Omega^2}\left(\frac{B}{2A}+1\right)\left\{\exp\left[-\frac{b(\omega_2^1+\omega_2^2)z}{2c_0^3\rho}\right]\right\} \tag{6}$$

where:

$$\omega_1=2\pi f_1;\ \omega_2=2\pi f_2;\ \Omega=2\pi F$$

The changes in the pressure level of primary wave, and of the sum- and difference-frequency wave obtained on the basis of the solution of the Burgers equation are shown in Fig. 2 .

Because of absorption increasing together with increasing frequency the second harmonic of primary waves disappears sooner then the wave of frequency $F=f_1-f_2$. Therefore outside of the area of interaction of primary waves limited by the distance of their attenuation the only wave that propagates is the one of frequency equal to the difference of the frequencies of the primary waves called a differencefrequency wave.

3. Features of parametric sources

In comparison with classical sources such source offers several advantages. Its characteristics features are highly directive broadband low-frequency transmission allowing fluent regulation of frequency of emitted wave, that is the difference between frequencies of primary waves, with physically small apertures (in comparison with wave lengths of the difference-frequency wave), and significantly reduced sidelobe. The main disadvantage of parametric sources is the low efficiency.

Typical directivity patterns of parametric array, i.e., the directivity pattern of primary wave and directivity pattern of difference frequency wave, are shown in Fig. 3. Directivity pattern of the wave of frequency equal to difference of frequencies of primary waves radiated when applying a linear transducer of the same area as a piezoceramic transducer used in a parametric array is shown for comparison.

The directivity of this pattern does not depend on dimensions of transducers radiating primary waves but depends on phenomena occuring in area of interaction. The directivity pattern of parametric array has no sidelobes, which are typical for linear transducers. The reason for this is that energy of primary waves is transferred to the secondary wave (difference frequency wave) only in high intensity in other directions is too low to cause originating of nonlinear effects.

Parametric gain, as a characterictic feature of nonlinear sources, was introduced to estimate effectiveness of transferring energy of primary waves to the difference frequency wave along the beam axis. It is defined as a complex function given by the following formula [16]:

$$g=\frac{Rp_F(R,\ \Theta=0)}{R_0p_{1,2}}=|g|\,e^{i\beta} \tag{7}$$

or in the logarithmic scale:

$$G=20\log|g| \tag{8}$$

where R is distance from transmitting transducer, R_0 is Rayleigh length ($R_0=S_0/\lambda$, S_0 is area of the transducer, λ is length of primary waves), $p_{1,2}$ is pressure amplitude of one of primary waves close to the source, $p_F(R,\Theta=0)$ is pressure amplitude of difference frequency wave.

Parametric gain G expresses in dB the difference between source level of difference frequency wave and source level for one of primary waves whereas phase angle indicates the area where difference frequency wave is mainly produced. Phase angle close to zero proves that nonlinear interactions take place mainly in the near field, yet its value close to $-90°$ corresponds to originating of difference frequency wave mainly in the far field.

4. Nonlinear properties of sea water

The scattering of sound by sound is the consequence of the nonlinearity of the medium.

The nonlinearity parameter B/A is regarded as important since it influences the process of generation of the secondary waves. Amplitude of difference-frequency wave is dependent on the value of parameter B/A.

The values of the parameter B/A for sea water, reported in literature, were obtained predominantly for oceanic highsalinity water [8]. Information about nonlinear properties of the Baltic Sea is required in application of nonlinear acoustic theory to its specific conditions. Yearly changes of the parameter B/A has been determined by means of the thermodynamic method. To verify the data obtained in this way a series of experimental investigations by means of the acoustical method were carried out at several points.

The thermodynamic method exploits the thermodynamic equations of the medium and data obtained by measuring the

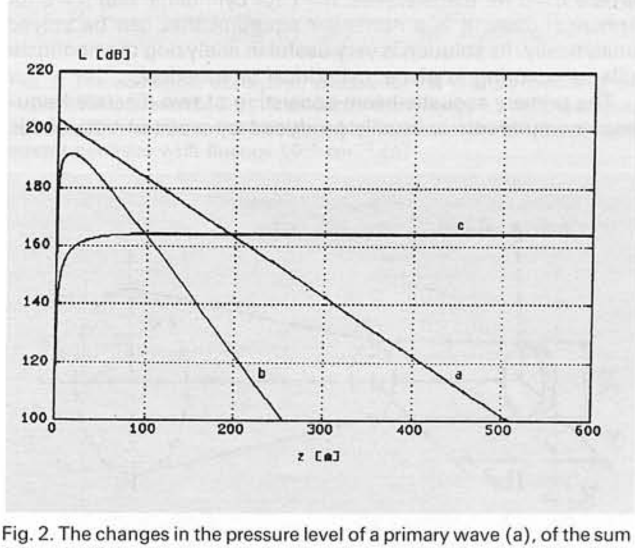

Fig. 2. The changes in the pressure level of a primary wave (a), of the sum (b) and difference-frequency (c) as a function of the distance from the source

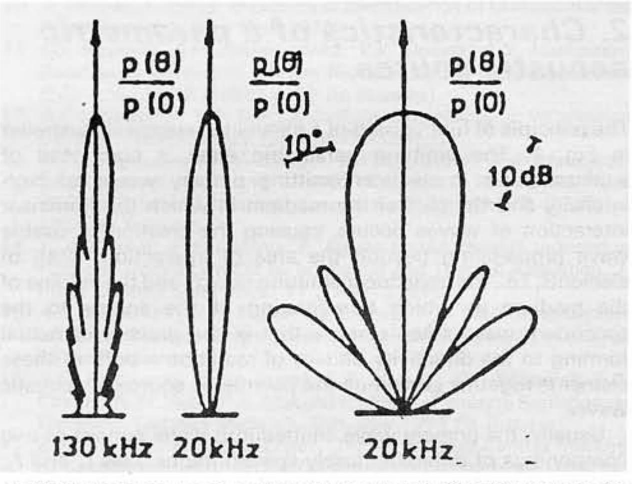

Fig. 3. Typical patterns of a parametric source: a) directivity pattern for a primary wave; b) directivity pattern for a difference frequency; c) directivity pattern for the wave of frequency equal to the difference of frequencies of primary waves radiated when applying a linear transducer of the same area as a piezoceramic transducer used in a parametric array

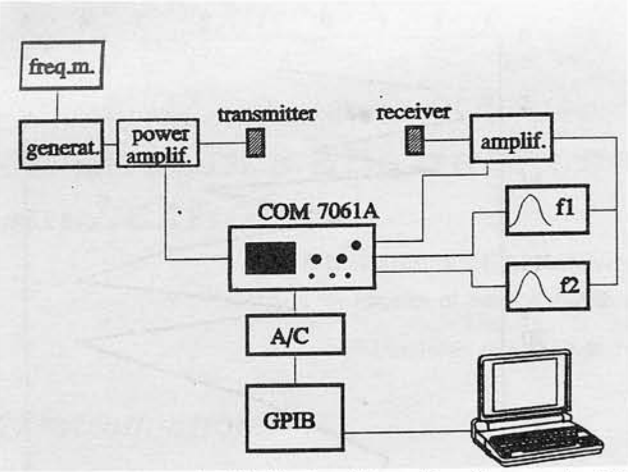

Fig. 4. The average vertical distribution of the nonlinearity parameter B/A in the years 1958–1993 at selected stations in the South Baltic Sea region; $B1$ – the Bornholm Deep, $B2$ – the Slupsk Furrow, $G2$ – the Gdansk Deep

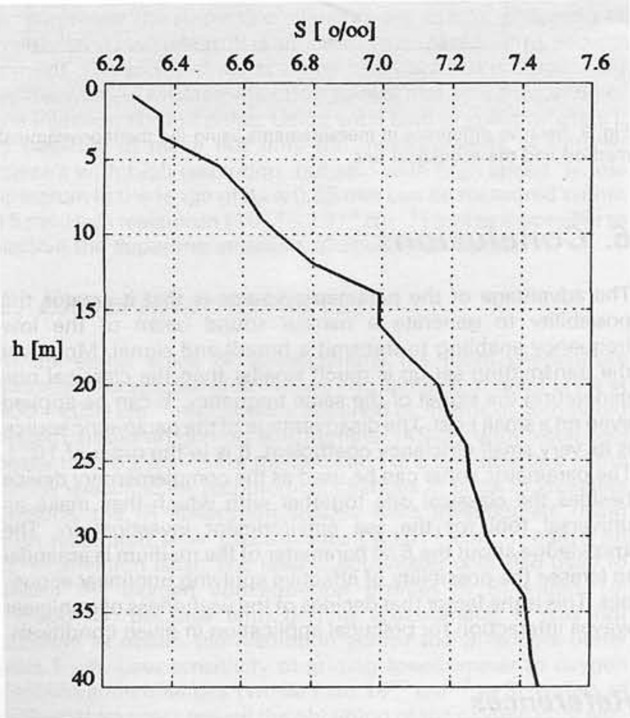

Fig. 6. Vertical distribution of salinity at the point of measurements

changes of temperature and salinity in the function of depth (the static pressure). The formula for B/A used in computation was introduced by R.T. Beyer [3] in following form:

$$\frac{B}{A} = 2\rho_0 c_0 \left(\frac{\delta c}{\delta p}\right)_{T,\rho=\rho_0} + \frac{2c_0\alpha T}{C_p}\left(\frac{\delta c}{\delta T}\right)_{p,\rho=\rho_0} \tag{9}$$

where: T is the absolute temperature, C_p is the specific heat at constant pressure, $\alpha=(1/V)(*V/*t)_p$ is the thermal expansion coefficient. Yearly changes in the nonlinearity parameter for selected stations in the South Baltic. Sea region are shown in Fig.4 in the average vertical distribution form.

The experimental investigation were carried out using the device for measuring the value of the nonlinearity parameter *B/A in situ* by means of acoustical method. A block diagram of the device is shown in Fig. 5. The piezoelectric transducer in circular piston form used as the transmitter radiates the monochromatic wave of frequency about 1 MHz. The circular piston of the same area as the transmitter is used as the receiver. The transducers are placed at the distance of about 0.2 m in such a way that the beam axes of both are coincident.

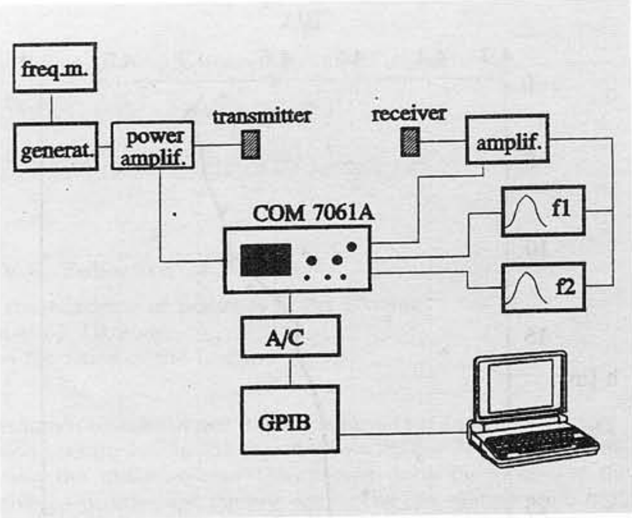

Fig. 5. The scheme of the measuring set up for the investigation of nonlinear wave-form distortion

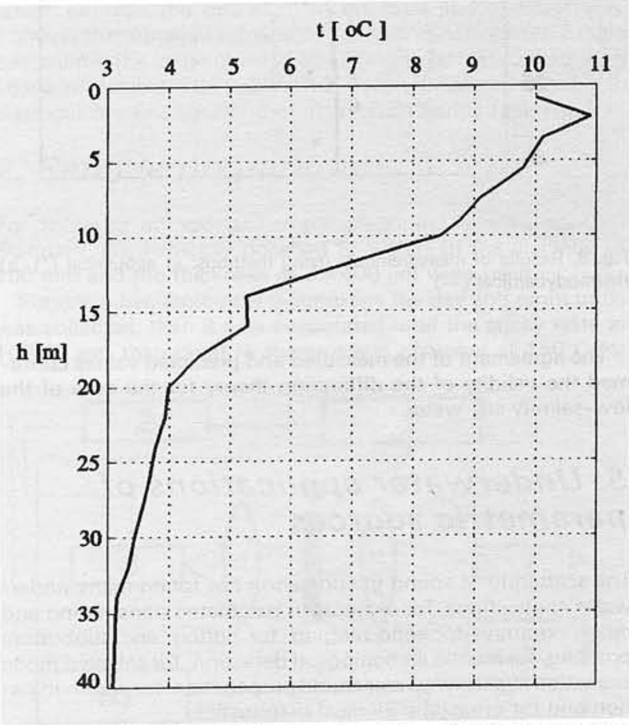

Fig. 7. Vertical distribution of temperature at the point of measurements

The value of the parameter B/A will be determined by means of separating the second harmonic component from the distorted signal and subsequently applying in calculations formula given by Cobb [7]:

$$|p_2(z)| = \rho_0^2\left(\frac{B}{2A}+1\right)\frac{k}{2\rho_0 c_0^2}|I_1 - I_2| \tag{10}$$

Fine structure in the near field of a radiated piston caused by diffraction was taken into account in the relation. The term I_1 takes into account dissipation losses and the term I_2 introduces a correcting element to the plane wave theory. It takes into account the diffraction spreading and phase cancellation over the receiver.

An example of the data obtained during the measurements together with the theoretically predicted curves are presented in the Figs. 8 and 9. In this case the investigations were carried out in the Slupsk Furrow region. The vertical distribution of the temperature and salinity of water at the place of measurements is shown in Figs. 6 and 7. The relative difference in measurements obtained using both of these methods are less than 4%.

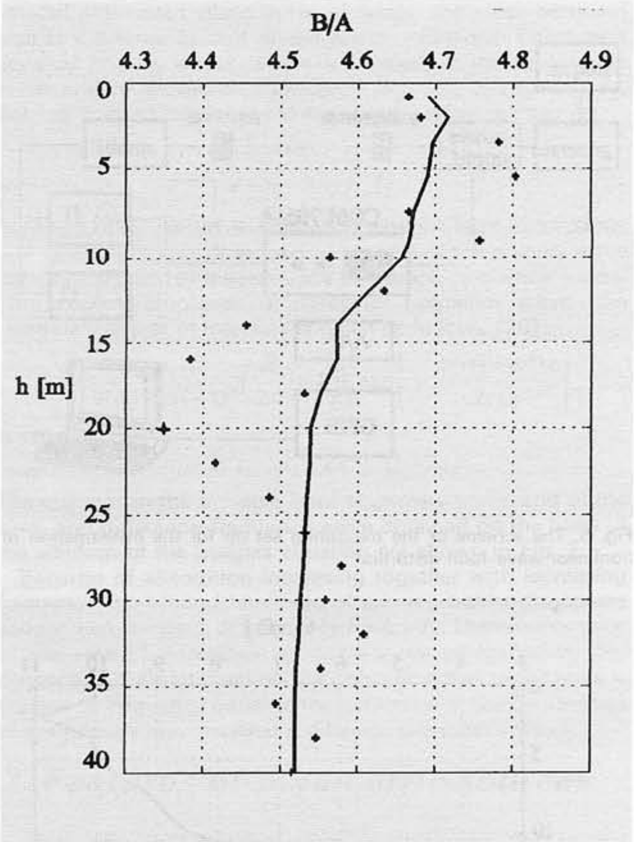

Fig. 8. Results of measurements using methods: a) acoustical (*), b) thermodynamical (—)

The agreement of the measured and predicted values confirmed the validity of the diffraction theory for the case of the low-salinity sea water.

5. Underwater applications of parametric sources

The scattering of sound phenomenon has found many underwater applications, for example in parametric transmitting and receiving arrays for echo ranging, for bottom and subbottom profiling, for marine archeological detection, for selected mode excitation in shallow water sound propagation, for communication and for ultrasonic medical diagnostics.

The parametric sources are used for investigations of the acoustic coditions in the ocean. The low-frequency measuring parametric system for the investigation in shallow water on the shelf has been described by T.G. Muir et al. [18]. The primary wave frequency was in the range of 12.5 kHz and the difference of frequencies of the primary wave components varied between 500 Hz and 5 kHz.

The highly directional acoustic beam produced by the parametric source is usually utilized for the penetration into sediments. The results of experimental study has been presented by T.G. Muir et al. [17]. Two different sources were used both of 20 kHz: the parametric source with a beam width about 2° and the linear source with a beam width about 10°. When the angle of the incident wave is near the critical angle, the parametric source generates a wave which propagates more steeply into the bottom than the wave generated by the linear source. A displacement of the parametric beam upon its transmission through the sediment interface was observed at the critical angle. The parametric devices for the investigation of the bottom structure have been more efficient and precise up till now. Their development allows the investigation of the sediment structure and a precise determination of the acoustic characteristics of the bottom materials.

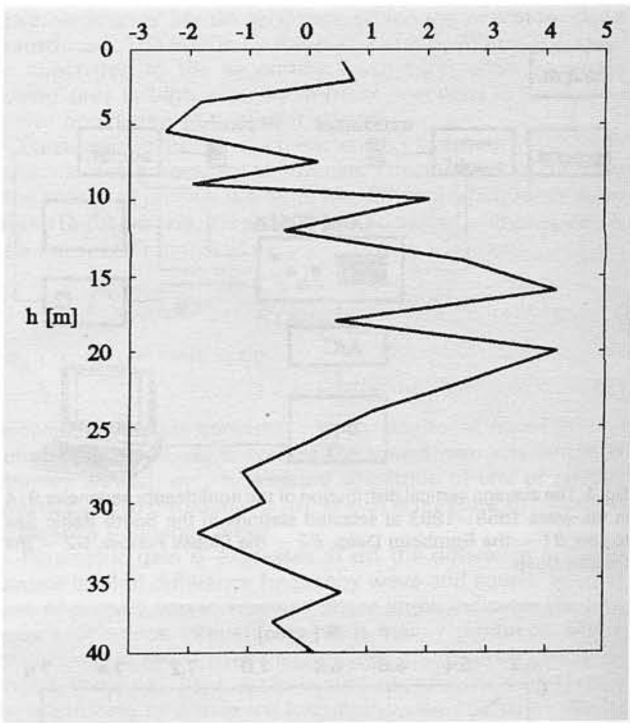

Fig. 9. Relative difference in measurements using the thermodynamical method and the acoustical one

6. Conclusions

The advantage of the parametric source is that it creates the possibility to generate a narrow sound beam of the low frequency enabling to transmit a broadband signal. Moreover the transmitting set up is much smaller than the classical one generating the signal of the same frequency. It can be applied even on a small boat. The disadvantage of the parametric source is its very small efficiency coefficient. It is in the order of 10^{-3}. The parametric sonar can be used as the complementary device besides the classical one together with which they make an universal tool for the sea einvironment investigation. The knowledge about the B/A parameter of the medium is essential to feresee the possibility of effective applying nonlinear acoustics. This is the factor that decides of the usefulness of nonlinear waves interaction for practical application in given conditions.

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