Contributed paper

Decomposition of Jones matrix on the quaternions field and its application in fiber components modeling

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Some parts of quaternions theory with the expansion to the matrix quaternions and their application in fiber components modeling are given in the paper. Decomposition of the Jones matrix to the quaternions enables a separate analysis of linear and circular birefringence and their influence on the fiber properties.

1. Introduction

The utilization of single-mode fibers and fiber components in systems, where it is necessary to protect the defined optical wave propagation, require suitable methods of description of these components, and analysis of their properties from the point of view of optical wave transmission and different affecting physical influences. This concerns coherent communication systems and information processing as well as interferometric sensor systems.

For description of transient properties of single-mode fiber components and design of single-mode fiber systems, utilization of Jones matrices, and for more precise description also scattering matrices is suitable [1, 2]. Some problems can occur if we measure parameters of Jones matrix elements. These parameters can be measured only in the defined geometrical arrangement of the fiber component. We can fulfil this condition by the analysis of component properties during defined physical influences and by the study of their effect.

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The effect of external physical influences was investigated in some previous works [4, 5, 8]. These were physical analyses answering the questions how the fiber arrangement and external influences (deformation) affect the fiber properties.

Useful results can also be achieved if we find the connection of these physical effects with the Jones matrix which describes the transfer properties of fiber components from the point of view of the fiber systems design.

The mathematical apparatus for decomposition of the Jones matrix, given in this paper, enables the analysis of physical effects on the linear and circular birefringence of fiber components. This decomposition would give not only mathematical conditions of unambiguity and complexity, but also the possibility of physical interpretation.

For decomposition we have used an apparatus of quaternions algebra which can be expanded to the matrix quaternions [3]. The basic knowledge on matrix quaternions algebra, its application in the Jones matrix decomposition, coupling to the coherent matrix of eigenvectors of Jones matrix and Stokes vectors, as well as application of quaternions for solution of coupling equations are given in this paper.

2. Decomposition of 2x2 matrices on the quaternions field

The quaternions create a system of generalization of complex numbers. Quaternion is commonly defined as:

$$x = a + \mathbf{i}b + \mathbf{j}c + \mathbf{k}d \tag{1}$$

where a, b, c, d are: real numbers; while $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are quaternions.

In equation (1) a represents the scalar part and $\mathbf{i}b+\mathbf{j}c+\mathbf{k}d$ the vector part of the quaternion.

Some laws are valid in the set of quaternions. In order to assure that the set of quaternions represent the field, the multiplicative law must be associative and distributive to the sum. For this presumption rules for multiplication of quaternions must be given, as follows:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{ijk} = -1 \tag{2}$$

$$\mathbf{ij} = -\mathbf{ji} = \mathbf{k},\ \mathbf{jk} = -\mathbf{kj} = \mathbf{i},\ \mathbf{ki} = -\mathbf{ik} = \mathbf{j} \tag{3}$$

To each quaternion (1) a symmetrical (inverse) quaternion exists defined by

$$x^{-1} = m^{-2}\sqrt{a^2 + b^2 + c^2 + d^2} \tag{4}$$

where $$m = \sqrt{a^2 + b^2 + c^2 + d^2} \tag{5}$$

is called the norm of quaternion.

An arbitrary quaternion (1) can be imagined as the set of real numbers and a vector $\mathbf{w}$. Commonly it is given, that to each vector $\mathbf{w}$ $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ we can append the vector quaternion $\mathbf{w} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$. This relation can be used in the tasks of rotation. Quaternion's algebra can be also extended to the matrix modification [3]. Consider the spin matrix given by the relations:

$$\mathrm{S}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \mathbf{S}_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix},\ \mathbf{S}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{6}$$

where i is the imaginary unit, and unit matrix is given as:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Linear matrix combination $\mathbf{I}$, $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ with real coefficients makes the modification of quaternions algebra. The scalar part of quaternion corresponds to the multiple of matrix $\mathbf{I}$ and the vector part to the linear combination of corresponding matrices $\mathrm{i}\mathbf{S}_1$, $\mathrm{i}\mathbf{S}_2$, $\mathrm{i}\mathbf{S}_3$. There is given:

$$\mathbf{S}_1^2 = \mathbf{S}_2^2 = \mathbf{S}_3^2 = \mathbf{I} \tag{7}$$

$$\begin{aligned} \mathbf{S}_2\mathbf{S}_3 &= -\mathbf{S}_3\mathbf{S}_2 = \mathrm{i}\mathbf{S}_1 \\ \mathbf{S}_2\mathbf{S}_1 &= -\mathbf{S}_1\mathbf{S}_3 = \mathrm{i}\mathbf{S}_2 \\ \mathbf{S}_1\mathbf{S}_2 &= -\mathbf{S}_2\mathbf{S}_1 = \mathrm{i}\mathbf{S}_3 \end{aligned} \tag{8}$$

Common matrix 2 x 2 can be decomposed to the quaternions, and corresponding coefficients can be derived:

$$\mathbf{J} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \xi_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \mathrm{i}\left(\xi_1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \xi_2 \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} + \right. \\ \left. + \xi_3 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right) \tag{9}$$

For ξ_i we have

$$\begin{gathered} \xi_0 = \frac{a+d}{2},\ \xi_2 = \frac{b-c}{2} \\ \xi_1 = -\mathrm{i}\frac{b+c}{2},\ \xi_3 = -\mathrm{i}\frac{a-d}{2} \\ \xi_0{}^2 + \xi_1{}^2 + \xi_2{}^2 + \xi_3{}^2 = 1 \end{gathered} \tag{10}$$

The relation (9) can also be written as

$$\mathbf{J} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \xi_0\mathbf{I} + \mathrm{i}\mathbf{H} \tag{11}$$

where

$$\mathbf{H} = \begin{bmatrix} \xi_3 & \xi_1 - \mathrm{i}\xi_2 \\ \xi_1 + \mathrm{i}\xi_2 & -\xi_3 \end{bmatrix} \tag{12}$$

Matrix $\mathbf{H}$ can be unambiguously appended to the definite vector. If the orthonormal base $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ is given, then to each real vector $\mathbf{x} = \xi_1\mathbf{u}_1 + \xi_2\mathbf{u}_2 + \xi_3\mathbf{u}_3$ the matrix $\mathbf{H}$ can be appended as:

$$\mathbf{H} = \begin{bmatrix} \xi_3 & \xi_1 - \mathrm{i}\xi_2 \\ \xi_1 + \mathrm{i}\xi_2 & -\xi_3 \end{bmatrix} = \xi_1\mathbf{S}_1 + \xi_2\mathbf{S}_2 + \xi_3\mathbf{S}_3 \tag{13}$$

From equation (13) we can see that the spin matrices $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ correspond to the base vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$.

Decomposition of the Jones matrix to the quaternions has a deep physical sense. Matrices $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ are the Jones matrices of the retarder $\lambda/2$ with the following meanings:

$\mathbf{S}_1$ – linear retarder $\lambda/2$ with the angle 45° to the axis x,

$\mathbf{S}_2$ – nonlinear (circular) retarder $\lambda/2$ with right polarization (45°),

$\mathbf{S}_3$ – linear retarder $\lambda/2$ with the angle 0° (90°) to the axis x.

Matrix $\mathbf{I}$ is the Jones matrix of free space.

From the given results we can see that the Jones matrix can be decomposed unambiguously and fully to the four elementary matrices whose linear combination creates the quaternion. This decomposition enables not only deeper analysis of the Jones matrices, but also it allows to find the effects which external influences on the component exerts on the Jones matrix of this component. At first we will analyze some common properties of the decomposition. We will calculate the eigenvalues and eigenvectors of the matrix $\mathbf{J}$ and $\mathbf{H}$ respectively.

The eigenvalues of matrix $\mathbf{H}$ are given by

$$\lambda_{1,2} = \pm \mathrm{i}\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2} = \pm \mathrm{i}\sqrt{D}, \qquad (14)$$

and the eigenvalues of matrix $\mathbf{J}$ by:

$$\lambda_{1,2} = \xi_0 \pm \mathrm{i}\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2} = \xi_0 \pm \mathrm{i}\sqrt{D}, \qquad (15)$$

The eigenvectors of matrices $\mathbf{J}$ and $\mathbf{H}$ are:

$$\mathbf{A} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\xi_1 - \mathrm{i}\xi_2}{-\xi_3 + \sqrt{D}} \\ 1 \end{bmatrix} \qquad (16)$$

$$\mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\xi_1 - \mathrm{i}\xi_2}{-\xi_3 - \sqrt{D}} \\ 1 \end{bmatrix} \qquad (17)$$

The product of $\mathbf{AA}^+$, where $\mathbf{A}^+$ is hermitian conjugate of the matrix $\mathbf{A}$, gives the coherent matrix with eigenvectors $\mathbf{A}$, $\mathbf{A}^*$, where asterix denotes the complex conjugate. Coherent matrix is given by the relation:

$$\mathbf{C} = \mathbf{AA}^+ = \begin{bmatrix} A_1A_1^* & A_1A_2^* \\ A_2A_1^* & A_2A_2^* \end{bmatrix} \qquad (18)$$

By the decomposition of the coherent matrix to the quaternions, we deduce that the coefficients of decomposition are the Stokes parameters corresponding to the eigenvectors:

$$\mathbf{C} = I.\mathbf{I} + \mathrm{i}\,(U\mathbf{S}_1 + V\mathbf{S}_2 + Q\mathbf{S}_3) \qquad (19)$$

where I stays for the optical wave intensity, Q, and U, V are the Stokes parameters of optical wave, described by the eigenvector of the Jones matrix.

For the further analysis, for example, for analysis of measurement results concerning the turned component, it will be useful to know how the decomposition to quaternions changes with rotation of the component by the angle Φ. The Jones matrix $\mathbf{J}_\Phi$ we obtain, from the matrix $\mathbf{J}$ of the considered component in the initial position, by the transformation

$$\mathbf{J}_\Phi = \mathbf{R}^T\mathbf{J}\mathbf{R} \qquad (20)$$

where

$$\mathbf{R} = \begin{bmatrix} \cos\Phi & \sin\Phi \\ -\sin\Phi & \cos\Phi \end{bmatrix} \qquad (21)$$

and $\mathbf{R}^T$ is matrix transposition of $\mathbf{R}$.

If the decomposition of matrix $\mathbf{J}$ contains the factors ξ_0, ξ_1, ξ_2, ξ_3 according to Equ. (9), the matrix $\mathbf{J}_\Phi$ can be decomposed as

$$\mathbf{J}_\Phi = x_0\mathbf{I} + \mathrm{i}\,(x_1\mathbf{S}_1 + x_2\mathbf{S}_2 + x_3\mathbf{S}_3) \qquad (22)$$

where

$$\begin{gathered} x_0 = \xi_0 \\ x_1 = \xi_1 \cos 2\Phi + \xi_3 \sin 2\Phi \\ x_2 = \xi_2 \\ x_3 = \xi_3 \cos 2\Phi + \xi_1 \sin 2\Phi \end{gathered} \qquad (23)$$

From the given equations one may see, that ξ_0 and ξ_2 are invariant to the rotation of the component. This corresponds to physical situation in which the Jones matrix of free space as well as of the component with circular birefringence does not change if the component turns.

3. Fiber modal analysis

Single-mode fiber properties can be described by the Jones matrix of common elliptical phase retarder [1].

$$\mathbf{J} = \begin{bmatrix} e^{i\frac{\delta}{2}}\cos^2\chi + e^{-i\frac{\delta}{2}}\sin^2\chi & ie^{-i\gamma}\sin 2\chi \sin\frac{\delta}{2} \\ ie^{i\gamma}\sin 2\chi \sin\frac{\delta}{2} & e^{-i\frac{\delta}{2}}\cos^2\chi + e^{i\frac{\delta}{2}}\sin^2\chi \end{bmatrix} \tag{24}$$

The two elliptically polarized modes with corresponding parameters χ, γ, δ can propagate in this phase retarder. Parameter χ is the ratio of x and y electric field intensity components, γ is the phase difference of these components and δ stays for the phase shift, due to different velocity of modes propagation.

By the modification of (24) we obtain the matrix:

$$\mathbf{J} = \begin{bmatrix} R_{11}\, e^{i\varphi} & iR_{12}\, e^{-i\gamma} \\ iR_{12}\, e^{i\gamma} & R_{11}\, e^{-i\varphi} \end{bmatrix} \tag{25}$$

where

$$R_{11} = \sqrt{\cos^2\frac{\delta}{2} + \cos^2 2\chi \sin^2\frac{\delta}{2}} \tag{26}$$

$$R_{12} = \sin 2\chi \sin\frac{\delta}{2} \tag{27}$$

$$\varphi = \operatorname{arctg} \cos 2\chi \operatorname{tg}\frac{\delta}{2} \tag{28}$$

With regard to the fact that matrices (24) and consequently (25) are normed, we can write

$$R_{11}^2 + R_{12}^2 = 1 \tag{29}$$

After modification of (25) we obtain

$$\mathbf{J} = e^{i\frac{\pi}{2}} \begin{bmatrix} R_{11}\, e^{i(-\frac{\pi}{2}+\varphi)} & R_{12}\, e^{-i\gamma} \\ R_{12}\, e^{i\gamma} & R_{11}\, e^{-i(\frac{\pi}{2}+\varphi)} \end{bmatrix} \tag{30}$$

If we mark $-\pi/2 + \varphi = -\varphi_{11}$ and $\gamma = \varphi_{12}$, where φ_{11} and φ_{12} will be the values of phases from the measurement, then we obtain:

$$\mathbf{J} = e^{i\frac{\pi}{2}} \begin{bmatrix} R_{11}\, e^{-i\varphi_{11}} & R_{12}\, e^{-i\varphi_{12}} \\ R_{12}\, e^{i\varphi_{12}} & -R_{11}\, e^{i\varphi_{11}} \end{bmatrix} \tag{31}$$

From this decomposition we find coefficients for decomposition to the quaternions

$$\begin{aligned} \xi_0 &= R_{11}\sin\varphi_{11} \\ \xi_1 &= R_{12}\cos\varphi_{12} \\ \xi_2 &= R_{12}\sin\varphi_{12} \\ \xi_3 &= R_{11}\cos\varphi_{11} \end{aligned} \tag{32}$$

Equations (31) and (32) are valid for the case of ideal retarder. In the case of a real measured retarder, errors appear during the process of determination of modules R_{ik} and phases φ_{ik}. With respect to the fact that matrix of an ideal retarder can be determined for arbitrary values of χ, δ, γ, we can find the matrix of ideal retarder corresponding to the measured matrix by the application of some iterative method. After that we can use relations (31) and (32).

From the given analysis the following conclusion is obvious. Fiber could be imagined as a common retarder with elliptical eigenmodes propagating with different velocities. The Jones matrix of this retarder can be decomposed to the linear combination of four elementary matrices: 1) matrix of free space, 2) matrix of half-wave linear retarder with the angle of fast axis 0° or 90°, 3) matrix of half-wave linear retarder with the angle of fast axis 45°, and 4) half-wave circular retarder with clockwise polarization.

A deeper analysis of fiber properties needs the study of polarization effect along the fiber by the coupling between two polarization modes.

We can use commonly known coupling equations:

$$\frac{d}{dz}\begin{bmatrix} E_{x0} \\ E_{y0} \end{bmatrix} = -i \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} E_{xl} \\ E_{yl} \end{bmatrix} \tag{33}$$

where E_x, E_y are elements of the Jones vector. Values of coupling coefficients N_{ik} can be determined for the cases of linear, circular or elliptical birefringence [8]. When considering the case of elliptical birefringence caused by the combination of torsion and transversal mechanical deformation, it is necessary to emphasize that this problem is not the case of torsioned birefringent fiber where the situation is more complicated. For our case we can write the relation for the coupling matrix $\mathbf{N}_e$ as follows:

$$\mathbf{N}_e = -i\begin{bmatrix} \beta_x & i\alpha \\ -i\alpha & \beta_x \end{bmatrix} \tag{34}$$

where α is the circular birefringence, β_x, β_y are the propagation coefficients in axis x and y.

In the case when linear birefringence operates in axes turned by angle Φ to axis x, the matrix $\mathbf{N}_e$ will be in the form $\mathbf{N}_e^{\Phi}$ given by

$$\mathbf{N}_e^{\Phi} = -i\begin{bmatrix} \beta + \frac{\Delta\beta}{2}\cos 2\Phi & i\alpha + \frac{\Delta\beta}{2}\sin 2\Phi \\ -i\alpha + \frac{\Delta\beta}{2}\sin 2\Phi & \beta - \frac{\Delta\beta}{2}\cos 2\Phi \end{bmatrix} \tag{35}$$

where $\Delta\beta = \beta_x - \beta_y$, $\beta = \frac{\beta_x + \beta_y}{2}$.

The value α is according [8] given as:

$$\alpha = g\tau, \tag{36}$$

where $g = 0{,}075$ for quartz, and τ is the torsion in turns per meter of length.

Common equation (33) can be solved with respect to its possible decomposition to the quaternions. Let us make its modification using Equ. (11):

$$\frac{d}{dz}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \left(\xi_0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + i\begin{bmatrix} \xi_3 & \xi_1 - i\xi_2 \\ \xi_1 + i\xi_2 & -\xi_3 \end{bmatrix}\right)\begin{bmatrix} E_x \\ E_y \end{bmatrix} \tag{37}$$

where

$$\xi_0 = i\,\frac{N_{11} + N_{22}}{2}$$

$$\xi_1 = \frac{N_{12} + N_{21}}{2}$$

$$\xi_2 = i\,\frac{N_{12} - N_{21}}{2}$$

$$\xi_3 = \frac{N_{11} - N_{22}}{2}$$

These equations we solve for initial conditions $z = 0$, $E_x = E_{xI}$, $E_y = E_{yI}$ and get, thus, the electric field intensities of fiber with length z as result:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \left\{ e^{\xi_0 z}\cos\sqrt{Dz}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + i\,\frac{e^{\xi_0 z}}{\sqrt{D}}\, sin\,\sqrt{Dz} \begin{bmatrix} \xi_3 & \xi_1 - i\xi_2 \\ \xi_1 + i\xi_2 & -\xi_3 \end{bmatrix}\right\}\begin{bmatrix} E_{xI} \\ E_{yI} \end{bmatrix} \tag{38}$$

where

$$D = \xi_1^2 + \xi_2^2 + \xi_3^2$$

If we consider that $\xi_0 = i\,\frac{N_{11} + N_{22}}{2}$, then $e^{\xi_0 z}$ is a complex exponential function which can be covered by the complex, time variant, input function $e^{i(\omega t + \xi_0 z)}$.

From equation (37) it is clear, that decomposition to the quaternion $\mathbf{S}_0$ and vector quaternions $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ depends upon the length of the fiber according the parameter $\sqrt{D}$. Parameter $\sqrt{D}$ expresses the velocity of birefringent variations along the fiber. It determines the beat length z_z:

$$\sqrt{D}\, z_z = 2\pi \tag{39}$$

In the next part we show that parameter $\sqrt{D}$ can be called the elliptical birefringence. Quaternion decomposition enables us to analyze measured values of Jones matrix elements, from the point of view of the precision of measurement, or fiber properties analysis. The validity of measured Jones matrices elements can be verified by the measurement of fiber in two positions, turned each other by the angle Φ. From (23) folows that after rotation of fiber, the coefficients ξ_0, ξ_2 will stay constant. From the point of view of the fiber properties analysis we can consider the influence of linear and circular birefringence for the given orientation of fiber with the consequent analysis of the effect of outside physical influences on the linear and circular birefringence i.e. on coefficients ξ_1, ξ_2, ξ_3. But it should be known that the decomposition of Jones matrix needs not give yet the information about linear and circular birefringence. In the next step we have to analyze the variability of fiber properties with the length of fiber.

For some of the linear and circular, or commonly elliptical birefringence and for definite length of fiber which is equal to the multiple of beat length, the fiber behaves as free space. For components ξ_1, ξ_2, ξ_3 the relation (39) is valid. Also in this case we can use decomposition for the description of the variations of fiber properties according to the effect of outside influences, e.g., induced birefringence caused by the effect of mechanical deformation. In addition it is clear from Equ. (38) that these methods can be used also for analysis of fiber length variation effect, also the effect of mechanical deformation.

As an example we introduce fiber stressed by the torsion, characterized by the value of circular birefringence α and linear birefringence $\Delta ß$, developed by the force effecting in the direction with angle Φ according to the axis x. Equation (38) for this case can be written as:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \left[\begin{array}{l} \cos\left(\sqrt{\left(\frac{\Delta ß}{2}\right)^2 + \alpha^2}\right) \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \\ + i \dfrac{\sin\left(\sqrt{\left(\frac{\Delta ß}{2}\right)^2 + \alpha^2}\right) z}{\sqrt{\left(\frac{\Delta ß}{2}\right)^2 + \alpha^2}} \left\{ \frac{\Delta ß}{2} \sin 2\Phi \right. \\ \left. \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \alpha \begin{bmatrix} 1 & -i \\ i & 0 \end{bmatrix} + \frac{\Delta ß}{2} \cos 2\Phi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \end{array} \right] \begin{bmatrix} E_{xl} \\ E_{yl} \end{bmatrix} \tag{40}$$

Vector part of quaternion can be defined in the base of orthonormal 3-D space (Fig. 1). The coefficient K_E is then equal to:

$$K_E = \frac{\sin\left(\sqrt{\left(\frac{\Delta ß}{2}\right)^2 + \alpha^2}\right) z}{\sqrt{\left(\frac{\Delta ß}{2}\right)^2 + \alpha^2}} \tag{41}$$

If we consider that linear and circular birefringence are determined by $\Delta ß/2$ and α, then the relation $\varepsilon = \sqrt{\left(\frac{\Delta ß}{2}\right) + \alpha^2}$ describes the final elliptical birefringence.

From relation (40) folows that decomposition on the quaternions enables separate analysis of linear and circular birefringences in reletion to fiber properties with respect to its length or variation of length respectively. Equation (40) has been derived for a simple case of linear and circular birefringence combination. A further necessary step is the determination of the coupling between physical influences and corresponding birefringence components, given by the vector part of the quaternion.

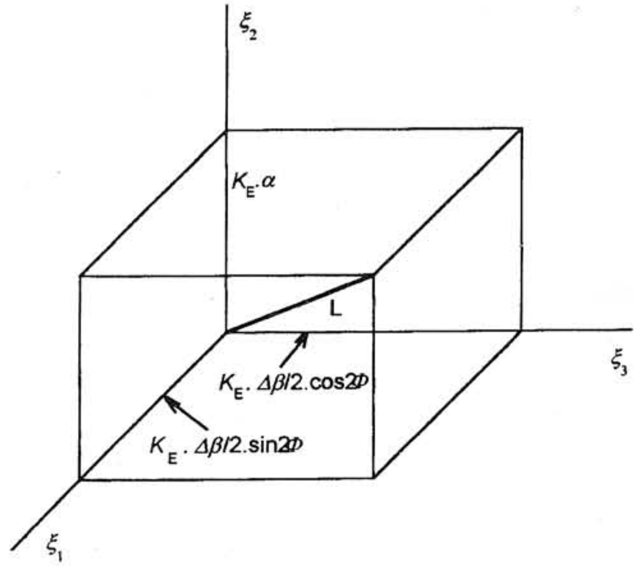

Fig. 1 Representation of vector part of quaternion (matrix **H**); $L = \sin\left(\sqrt{\left(\frac{\Delta ß}{2}\right)^2 + \alpha^2}\right) z$

On the basis of previous analysis, we summarize the basic properties of decomposition on the matrix quaternions.

1. Matrix quaternions create the field where it is possible to make complete and unambiguous decomposition of 2x2 matrix, the Jones matrix in our case.

2. Decomposition to quaternions enables separate analysis of linear and circular birefringence effects on birefringent properties of fiber component. Following with the knowledge about coupling between effecting physical quantity for example mechanical stress and birefringence components, this method enables an analysis of the effect which physical quantities exert on the resultant properties of fiber component.

3. Decomposition coefficients of coherent matrix of the Jones matrix eigenvectors on the quaternions are equal to the Stokes vectors. We can find

the mutual relation between the representation of these Stokes vectors on the Poincaré sphere and the representation of the vector part of the Jones matrix quaternion.

4. Decomposition to the quaternions enables us to analyze the variations of fiber properties with the length of fiber.

5. Decomposition to quaternions contributes to the evaluation of regularity and precision of the measured Jones matrix elements of the fiber component. For example, after fiber component turning, the components of decomposition ξ_0 and ξ_2, corresponding to the free space and circular birefringence can't change. In this sense the decompositon has been used for appreciation of measured results.

4. Fiber components measurement

The working place used for measurement is composed of three parts: 1) A source of light and components for an arrangement of the input optical beam, 2) the measured component in a defined condition with the coupling of light to the fiber, 3) the components for the transformation of the output optical beam. The set of instruments mentioned in points 1 and 3 is related to the method described in [1]. The results of the measurement for different fiber components are given in [7] and [10]

5. Conclusion

Decomposition to the quaternions enables a separate analysis of linear birefringence influence, which operates with some angle, and circular birefringence on the fiber properties with respect to its length or variations of length respectively. Matrix quaternions define the field, and on this field we can perform full and unambiguous decomposition of the Jones matrix 2x2. This decomposition enables also separate analysis of linear and circular birefringence influences on the birefringence properties of the fiber component. With an application of knowledge concerning coupling between the effecting physical quantity (mechanical strain) and the birefringence elements, this method enables analysis of physical quantity influence on the resultant properties of fiber component (in the case of sensor). Decomposition to quaternions contributes to the improvement of precision of the Jones matrices elements measurement.

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