Temporal coherence and mode structure of the He-Ne laser beam spectrum

PETR HLUBINA*

Institute of Physics, Silesian University at Opava, Czech Republic

Theoretical and experimental analysis of the partial temporal coherence of the He-Ne laser beams is performed which leads to the conclusion that only two longitudinal modes in the corresponding laser cavity oscillate. It is also revealed experimentally that the spectra of the two particular He-Ne lasers consist of two lines of a Lorentzian spectral profile and that the length of the laser cavity may be determined from the position of the first zero in the relation between the modulus of the complex degree of temporal coherence of the He-Ne laser beam and the optical path difference.

1. Introduction

The optical fields generated by light sources may be described from a statistical point of view. In the statistical sense, two extreme cases exist which are represented by incoherent and coherent sources. An incoherent or a chaotic source generates the optical field in a disordered manner; on the other hand, a coherent source generates the optical field in an ordered manner. It is clear that incoherent and coherent sources are only mathematical idealizations and the sources which we generally encounter in practice, that is the partially coherent sources, lie between these two extremes. A thermal source may serve as an example of a source which radiates in a highly disordered manner. Stabilized lasers radiate, on the other hand, in a highly ordered manner and that is a reason for using them in a great number of applications.

Since the discovery of the, at present most frequently used, He-Ne laser a need exists for the determination of the statistical properties of its radiation. Within the framework of classical coherence theory, the statistical properties of sources, which include He-Ne lasers, and of the partially coherent optical fields that they generate may be considered in the space-time domain. Numerous theoretical and experimental investigations [1-6] have revealed that the spatial coherence of the He-Ne laser beams is constrained by the presence and nature of the transverse modes giving the highest spatial coherence if the He-Ne laser radiates in a single transverse mode. Theoretical investigations of temporal coherence have revealed that He-Ne lasers are quasi-monochromatic sources and that the concept of longitudinal modes has to be adopted. Consequently, various methods exist which can be used to control a number of the longitudinal modes [7].

In this contribution, both theoretical and experimental analysis of the partial temporal coherence of the He-Ne laser beam is presented. The formalism of classical coherence theory is used to evaluate theoretically the complex degree of temporal coherence and its modulus. To this the

* address for correspondence: Petr Hlubina, Institute of Physics, Silesian University at Opava Bezrucovo nam 13, 746 01 Opava, Czech Republic, e-mail:
Petr. Hlubina - at - decsu.fpf.slu.cz

model of the spectrum of radiation of the He-Ne laser which consists of longitudinal modes, that is the eigenmodes of laser cavity, is adopted. The results of the theoretical approach are compared with those obtained experimentally using Fourier spectroscopy in a Michelson interferometer. From the measured dependence of the modulus of the complex degree of temporal coherence on the optical path difference it has been evidenced that the theoretical approach is suitable for the two particular He-Ne lasers with different lengths of laser cavities if only two longitudinal modes are taken into account. It is also revealed that the spectra of both He-Ne lasers consist of two lines of a Lorentzian spectral profile and that the length of the He-Ne laser cavity may be determined from the position of the first zero in the dependence of the modulus of the complex degree of temporal coherence of its radiation on the optical path difference.

2. Theoretical background

Within the framework of classical coherence theory the statistical properties of the sources and of the fields that they generate are described by coherence functions in the space-time or the space-frequency domain. Consider a random scalar field, represented by a statistical ensemble $\{V(\mathbf{r};t)\}$. The argument r denotes the position vector of a typical field point and t denotes the time. We take $V(\mathbf{r};t)$ to be the complex analytic signal representation of the real field and we assume that the field is stationary, at least in the wide sense, and ergodic [8−11]. The statistical properties of the field in the space-time domain are characterized by the second-order coherence function, that is by the mutual coherence function:

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \langle V^*(\mathbf{r}_1; t) V(\mathbf{r}_2; t+\tau) \rangle, \quad (1)$$

where the angular brackets denote the ensemble average, or under the above mentioned condition of the field ergodicity, the time average, t denotes the time delay and the asterisk denotes the complex conjugate. Consequently, the statistical properties of the field in the space-frequency domain are characterized by the cross-spectral density function:

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) exp(i\omega\tau) d\tau, \quad (2)$$

where ω is the angular frequency. The cross-spectral density function forms, according to the Wiener-Khintchine theorem [9−11], a Fourier transform pair with the mutual coherence function, i. e. the cross-spectral density function is spectrally decomposed:

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \int_0^{\infty} W(\mathbf{r}_1, \mathbf{r}_2; \omega) exp(-i\omega\tau) d\omega, \quad (3)$$

Consider now a simple interference experiment in which the field coherence in the space-time domain may be studied. In a two-beam interference experiment (see Fig. 1) two isolated beams from points $P1(r_1)$ and $P2(r_2)$ with pinholes on an

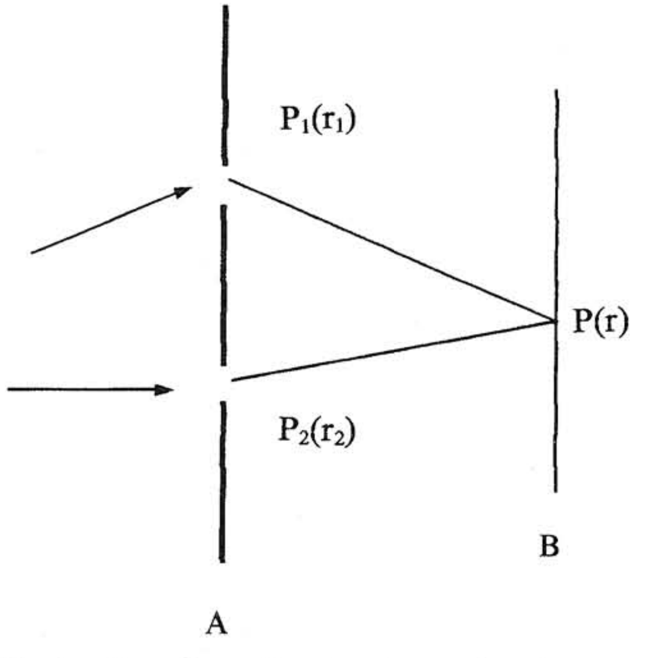

Fig 1. Illustration of a two-beam interference experiment.

opaque screen A are superposed at a point P(r) on the screen B. The averaged intensity of the light at P may be expressed in the form [9−11]:

$$I(\mathbf{r}; \tau) = I^{(1)}(\mathbf{r}) + I^{(2)}(\mathbf{r}) + \\ + 2\,[I^{(1)}(\mathbf{r}) I^{(2)}(\mathbf{r})]^{1/2} \mathrm{Re}\{\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)\}, \quad (4)$$

where $I^{(1)}(\mathbf{r})$ and $I^{(2)}(\mathbf{r})$, respectively, represent the averaged intensities of the light beams reaching the point $P(\mathbf{r})$ from the single points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ and where the complex degree of mutual coherence has been introduced by

$$\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)}{[\Gamma(\mathbf{r}_1, \mathbf{r}_2; 0)\ \Gamma(\mathbf{r}_1, \mathbf{r}_2; 0)]^{1/2}} \quad (5)$$

From Schwarz inequality it follows that the modulus of the complex degree of mutual coherence is constrained by the relationship:

$$0 \leqslant |\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)| \leqslant 1, \quad (6)$$

which is important from the point of view of the sharpness of the interference effects to which superposition of the two beams may give rise.

It may be shown [9−11] that in many cases of practical interest the complex degree of mutual coherence may be expressed, at least to a good approximation, as the product of two functions:

$$\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \gamma(\mathbf{r}_1, \mathbf{r}_2; 0)\gamma(\tau) \quad (7)$$

one of which, $\gamma(r_1, r_2; 0)$, is the complex degree of spatial coherence and the other, $\gamma(\tau)$, is the complex degree of temporal coherence, which is related to the source spectral density $S(\omega)$ through the relation:

$$\gamma(\tau) = \int_0^\infty S(\omega)\exp(-i\omega\tau)d\omega, \quad (8)$$

if the normalization condition

$$\int_0^\infty S(\omega)d\omega = 1 \quad (9)$$

applies. Relation (7) implies that the spatial and temporal coherences of the field are separable hence the concept of cross-spectral purity can be adopted.

We restrict ourselves now to a two-beam interference experiment with the Michelson interferometer in which only the temporal coherence of the field is studied. In this case $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$ and the interference law (4) may be rewritten in the form:

$$I(\mathbf{r}; \tau) = I^{(1)}(\mathbf{r}) + I^{(2)}(\mathbf{r}) + 2\,[I^{(1)}(\mathbf{r})I^{(2)}(\mathbf{r})]^{1/2}\,|\gamma(\tau)|\,\cos[\alpha(\tau) - \omega_0\tau], \quad (10)$$

where

$$\alpha(\tau) = \arg\gamma(\tau) + \omega_0\tau. \quad (11)$$

A measure of the sharpness of interference fringes in the two-beam interference of mutually delayed beams in the Michelson interferometer is the visibility. The visibility at a point $P(\mathbf{r})$ in an interference pattern is defined for the time delay τ as

$$V(\mathbf{r}; \tau) = \frac{I_{max}(\mathbf{r}; \tau) - I_{min}(\mathbf{r}; \tau)}{I_{max}(\mathbf{r}; \tau) + I_{min}(\mathbf{r}; \tau)} \quad (12)$$

On substituting from equation (10) into equation (12), we obtain

$$V(\mathbf{r}; \tau) = \frac{2\,[I^{(1)}(\mathbf{r})\ I^{(2)}(\mathbf{r})]^{1/2}}{I^{(1)}(\mathbf{r}) + I^{(2)}(\mathbf{r})}\,|\gamma(\tau)|, \quad (13)$$

from which it is clear that the modulus of the complex degree of temporal coherence is simply equal to the visibility of the interference fringes when the averaged intensities of the two beams are equal, that is $I^{(1)}(\mathbf{r}) = I^{(2)}(\mathbf{r}) = I(\mathbf{r})$, and it may thus be determined from simple measurements. This is the principle of the method of interference spectroscopy known as Fourier spectroscopy [11] because, from the measured dependence of the visibility $V(\mathbf{r}; \tau)$ on the time delay τ, one may obtain the source spectral density $S(\omega)$.

To clarify the advantages of this method we now consider the spectral profiles commonly used; a Lorentzian and a Gaussian spectral profile [9−11]. If the Lorentzian spectral profile centered at the angular frequency ω_0 with a halfwidth Γ at $1/2S(\omega_0)$ is considered, the spectral density is expressed as

$$S(\omega) = \frac{\Gamma/\pi}{(\omega-\omega_0)^2 + \Gamma^2}, \quad (14)$$

and for the complex degree of temporal coherence we may write

$$\gamma(\tau) = \exp(-\Gamma\,|\tau|\,)\exp(-i\omega_0\tau), \quad (15)$$

from which it is clear that $\alpha(\tau) = 0$, i. e. the spectrum is symmetrical. Similarly, if the Gaussian spectral profile centred at the angular frequency ω_0 with a halfwidth Γ at $e^{-1}S(\omega_0)$ is considered, the spectral density is expressed as

$$S(\omega) = \frac{1}{\sqrt{\pi}\Gamma}\exp\left[-\frac{(\omega - \omega_0)^2}{\Gamma^2}\right], \quad (16)$$

and by applying equation (8) we obtain, for the complex degree of temporal coherence:

$$\gamma(\tau) = \exp(-\frac{\Gamma^2}{4}\tau^2)\exp(-i\omega_0\tau). \qquad (17)$$

The modulus $|\gamma(\tau)|$ is for both spectral profiles monotonously decreasing with increasing τ, but in different, easily distinguished ways.

Now, we turn our attention to the spectral representation of the laser radiation. The laser model, which is usually adopted, operates with a laser cavity in which N longitudinal, equally separated modes oscillate. The frequency spacing ν_s between modes is [7]:

$$\nu_s = \frac{c}{2L}, \qquad (18)$$

where c is the velocity of light in the free space and L is the laser cavity length. If the relation (18) is rewritten into the relation in the angular frequency domain:

$$\Gamma_s = \pi\frac{c}{L}, \qquad (19)$$

we may take into account the model that is based on the Lorentzian spectral profile of individual modes. This means that the corresponding spectral density may be written as

$$S(\omega) = \frac{\Gamma}{\pi}\frac{1}{N}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{1}{(\omega - \omega_0 + n\Gamma_s)^2 + \Gamma^2}, \qquad (20)$$

and, on substituting equation (20) into equation (8), we obtain for the complex degree of temporal coherence:

$$\gamma(\tau) = \frac{1}{N}\sum_{n=-(N-1)/2}^{(N-1)/2}\exp(in\Gamma_s\tau)\times \exp(-\Gamma|\tau|)\exp(-i\omega_0\tau). \qquad (21)$$

Substantial differences between the dependences of the modulus of the complex degree of temporal coherence on the time delay are demonstrated in Fig. 2 and Fig. 3, respectively. This is valid for the source spectra with the same width at half of maximum and spacing between modes $\Gamma_s = 25\Gamma$ but with a different number of modes. Figures, in which only a positive time delay is taken into account, are related to a two-mode spectrum and three-mode spectrum, respectively. It should be emphasized that a number of modes in the source spectrum may be determined by a comparison of measured and theoretically determined positions of zero points in the dependence of modulus of the complex degree of temporal coherence on the time delay. In similar way the model with the Gaussian spectral profile of the individual modes may be considered.

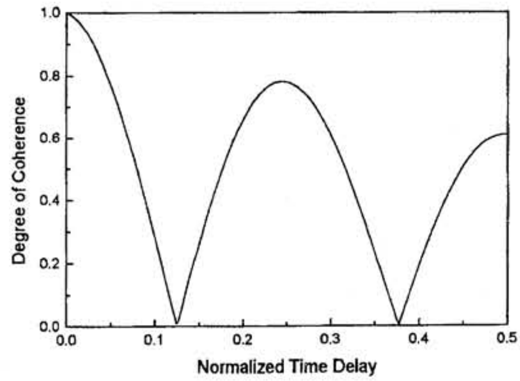

Fig. 2. Modulus of the complex degree of temporal coherence as a function of the normalized time delay for two individual modes of the Lorentzian spectral profile.

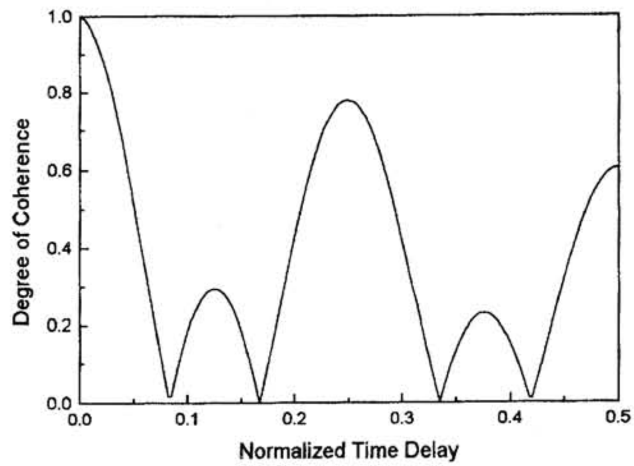

Fig. 3. Modulus of the complex degree of temporal coherence as a function of the normalized time delay for three individual modes of the Lorentzian spectral profile.

3. Experiment

The method of Fourier spectroscopy has been used in the determination of the mode structure of symmetrical spectra of a number of He-Ne lasers. First, the value of the visibility $V(\mathbf{r}; \tau)$ at the point $\mathbf{r}$ in the interference pattern was determined from the measured intensities $I_{min}(\mathbf{r}; \tau)$ and $I_{max}(\mathbf{r}; \tau)$ for the time delay τ of the two superposed light beams originated from the same source, the He-Ne laser. Next, the measured values of the beam intensities $I^{(1)}(\mathbf{r})$ and $I^{(2)}(\mathbf{r})$ were used for the determination of the modulus $|\gamma(\tau)|$ of the complex degree of temporal coherence by applying equation (13). The value of the time delay τ was varied and consequently the dependence of $|\gamma(\tau)|$ on τ was determined from which the spectral density $S(\omega)$ of the He-Ne laser was evaluated.

The experimental set-up (see Fig. 4) consisted of the Michelson interferometer and a detection part including electronics and a computer. The most important part was a PZT translator which moved with one of the two mirrors in the position for which the optical path difference $\Delta l = c\tau$ corresponds to, so that the interference pattern which consisted of several fringes could be analyzed by the computer (the values $I_{min}(\mathbf{r}; \Delta l)$, $I_{max}(\mathbf{r}; \Delta l)$, and consequently, $|\gamma(\Delta l)|$ were determined). The beam expander was also used for easy operation with light beams and the pinhole, respectively. It was used in front of the PIN photodetector in order to ensure the spatial integration. However, when the interference pattern was detected the expander was not present.

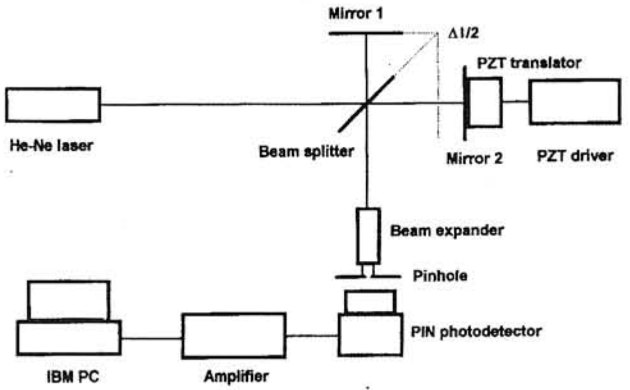

Fig. 4. Experimental set-up for the measurement of temporal coherence of the He-Ne laser beam.

4. Experimental results, discussion

A large number of low-power He-Ne lasers with two different laser cavity lengths were analyzed experimentally. However, the stable visibility dependence was achieved for only two He-Ne lasers.

The dependence of the modulus of the complex degree of temporal coherence $|\gamma(\Delta l)|$ on the optical path difference Δl is shown for the first He-Ne laser in Fig. 5, and for the second He-Ne laser in Fig. 6, respectively. It is clearly seen from both figures that the model of mode structure which operates with two longitudinal modes may be

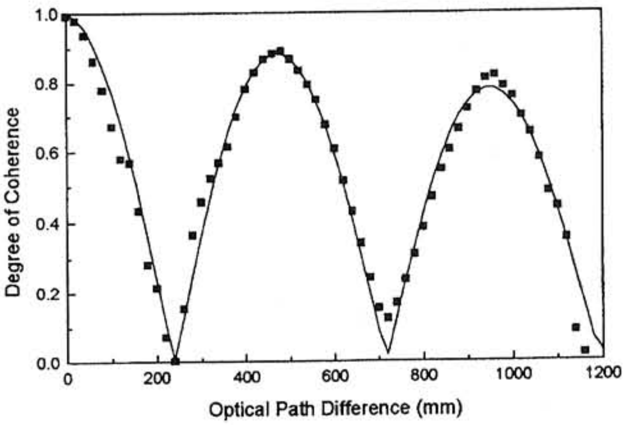

Fig. 5. Measured modulus of the complex degree of temporal coherence as a function of the optical path difference for the first He-Ne laser (markers), along with least-squares best fit (full curve).

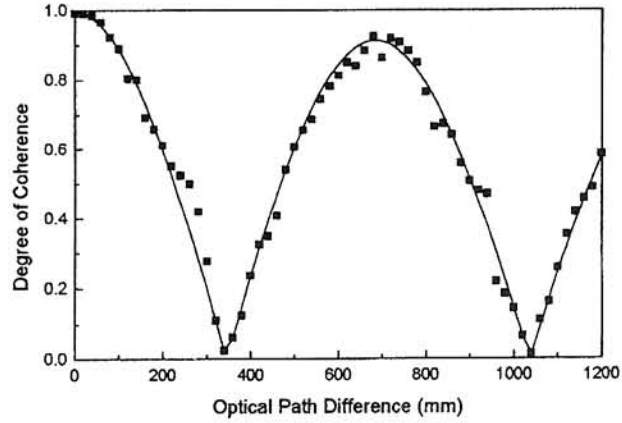

Fig. 6. Measured modulus of the complex degree of temporal coherence as a function of the optical path difference for the second He-Ne laser (markers), along with least-squares best fit (full curve).

adopted. By a least-squares procedure it was revealed that the model of the Lorentzian spectral profile of individual modes is more adequate than the model of the Gaussian spectral profile of individual modes. The relevant theoretical dependence results from the equation (21) and has the form:

$$|\gamma(\Delta l)| = \left|\cos\left(\frac{\Delta l}{l_s}\right)\right| \exp\left(-\frac{|\Delta l|}{l_c}\right), \qquad (22)$$

where $l_s = 2c/\Gamma_s$ and $l_c = c/\Gamma$. The theoretical dependence (22), with parameters l_s, l_c obtained by a least-squares procedure, is in good agreement with both measured dependencies (see Figs. 5 and 6). The values of the parameter l_s for both He-Ne lasers, or equivalently the values Δl_0 for the first zero in the dependence $|\gamma(\Delta l)|$ on Δl, may be used in the evaluation of the lengths of laser cavities: $L = l_s\pi/2$ or equivalently $L = \Delta l_0$. For the first He-Ne laser we have obtained L = 23.9 cm, while for the second He-Ne laser the obtained result was L = 34.5 cm. Both values of the laser cavity length are in good agreement with lengths of laser tubes (for the first laser it is 25.5 cm, and for the second laser it is 35 cm).

5. Conclusions

The feasibility of Fourier spectroscopy in determination of the temporal coherence of laser beams and consequently the mode structure of the spectra of He-Ne lasers has been confirmed. It was revealed that even if there is no possibility to use high resolution spectroscopy based on application, for example, of a scanning Fabry-Perot interferometer for determination of the source spectrum, the simple method of Fourier spectroscopy can be used and the mode structure as well as line shape of the source spectrum can be resolved.

We have shown that the spectra of two particular He-Ne lasers consist of two lines of a Lorentzian spectral profile. Consequently, the length of the corresponding laser cavity has been determined from the position of the first zero in the dependence of the modulus of the complex degree of temporal coherence of the He-Ne laser beam on the optical path difference.

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