# Investigation of optical elements in laser system: a plane parallel plate

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*The paper presents a simple method of qualification and verification of a plane-parallel optical element from the point of view of its application in different optical systems. The analyzed object is a plane-parallel plate of BK7 glass illuminated with a He-Ne laser beam. The analyzed examples can be easily generalized to other materials and wavelengths. Short wave and long wave absorption bands and dispersion of the glass refractive index have been omitted in order to simplify the problem.*

*The relevant equations expressing the spectral characteristics of the plane-parallel plate have been solved and the results of calculations have been compared with the results of measurements.*

*On the basis of the performed experiments it has been concluded that the measurement of the dependence of optical transmissivity on angle of incidence in the range from 0° to 10° makes possible a satisfactory estimation of the influence of a plane-parallel plate on the spectral distribution of the coherent beam. These measurements enable the determination of the fractional part of phase thickness of the plate for the applied wavelengths.*

## 1. Introduction

The plane-parallel plates are widely used as beam splitters, optical windows and substrates for thin multilayers. The application of isotropic, plane-parallel plate is a basic technique in the optics and was analyzed in detail earlier [1, 2]. The analysis was also made for laser radiation applications [3]. It is continued [4, 5] due to the new possible applications and new measuring techniques.

The interferencial methods belong to the most precise techniques of measurement of thickness. They afford an accuracy of $\gamma/100$ and even $\gamma/1000$ [6] when interferometer with a discrete change of phase on a special investigation post is applied.

The fabrication of optical elements is closely connected with parameter control at every stage of the treatment process. Before fabrication we need to know the characteristics of the raw material and its optical orientation. Then we should plan precisely the thickness of manufactured element. In some cases, especially when the elements are illuminated with laser radiation of high power density, it is important for personal safety and element durability. That is why the adequate attest of fabricated optical elements is requested. On the other hand the comprehensive measurements of elements are very expensive and labo-

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rious. To decrease the costs of measurement it is important to define the conditions in which every element will be used and to reduce the scope of measurements to those parameters which are fundamental for the selected applications. The test should be made in the same operating conditions as the real applications conditions.

This paper presents a simple method of qualification of the applied element and verification if its dimensions conform to requirements. The matter of the analysis is the possibly simplest optical element i.e. plane-parallel plate of BK7 glass illuminated with He-Ne laser beam. Analyzed examples can be easy generalized to other materials and wavelengths. Short wave and long wave absorption bands and dispersion of refractive index of glass have been omitted to simplify the problem.

## 2. Spectral characteristics of plane-parallel plate

A theoretical analysis of plane-parallel plate of BK7 glass of physical thickness d ($n_p = 1.52$; imaginary part and dispersion of refractive index are omitted) leads to the following formula, valid for both polarisation factors:

$$R_{(i)} = \frac{\left(\frac{n_{s(i)} \cdot n_{o(i)}}{n_{p(i)}} - n_{p(i)}\right)^2 \cdot \sin^2\varepsilon}{(n_{s(i)} + n_{o(i)})^2 \cdot \cos^2\varepsilon + \left(\frac{n_{s(i)} \cdot n_{o(i)}}{n_{p(i)}} + n_{p(i)}\right)^2 \cdot \sin^2\varepsilon} \tag{1}$$

where:

i – one of the linear polarized components, parallel or perpendicular to the plane of incident; type p or s,

– for plane-parallel plate:

$$n_{p(p)} = n_p/\cos\beta, \quad n_{p(s)} = n_p \cos\beta,$$

– for surrounding medium (side of incident radiation):

$$n_{o(p)} = n_o/\cos\alpha, \quad n_{o(s)} = n_o\cos\alpha,$$

– for surrounding medium (side of outgoing radiation):

$$n_{s(p)} = n_s/\cos\gamma, \quad n_{s(s)} = n_s\cos\gamma,$$

$\alpha$ – the angle of incident radiation on the plate in surrounding medium with refractive index $n_o$, $\beta$ and $\gamma$ are determined by Snell's law: $n_o\sin\alpha = n_p\sin\beta = n_s\sin\gamma$,

$\varepsilon = (4\pi d/\lambda)n_p \cos\beta \pm \pi$, the phase difference between the two waves (of wavelength $\gamma$) at the front and back surfaces.

For each polarization, the reflectivity ($R_{(i)}$) and transmissivity ($T_{(i)}$) = 1 – ($R_{(i)}$) are periodic functions of optical thickness of a plane-parallel plate. As a function of the angle of incidence, the periodic function falls within an envelope that is substantially different for each polarization.

In accordance with Eq. (1) the transmissivity of thin plane-parallel plate of BK7 glass of physical thickness d = 0.1 mm has been calculated for the radiation of wavelength 632.8 nm as a function of the angle of incidence. These transmissivity dependencies are presented in Fig.1. For incidence angles between 20° and 80° only envelopes are

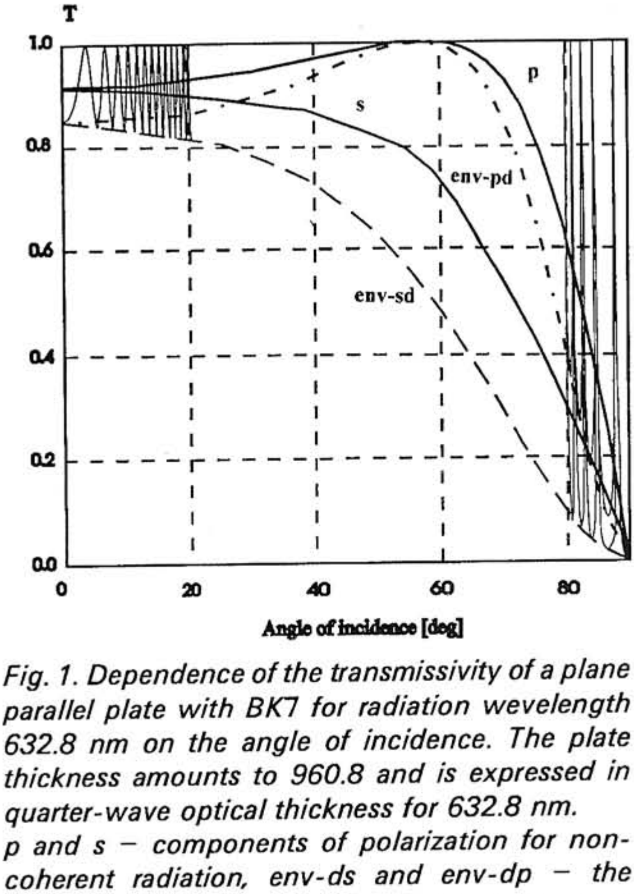

*Fig. 1. Dependence of the transmissivity of a plane parallel plate with BK7 for radiation wevelength 632.8 nm on the angle of incidence. The plate thickness amounts to 960.8 and is expressed in quarter-wave optical thickness for 632.8 nm. p and s – components of polarization for non-coherent radiation, env-ds and env-dp – the bottom envelopes respectively for s- and p- type polarization of coherent radiation. For angles of the incidence between 20° and 80° only envelopes have been presented.*

presented (for bottom values env-dp and env-ds for p and s polarization, respectively and for top value T = 1.0). The changes of both linear components of non-coherence radiation usually obtained from spectrophotometer measurements are presented for comparison. The results of spectrophotometer measurements are independent on the thickness of a plane-parallel plate in a wide range of tens of millimetres (when the glass used as a substrate is of high quality – without bubbles or other elements destroying homogeneity and isotropy of the substrate). Such values are typical for plane-parallel plates applied in optical systems.

Figs. 2–6 present the transmissivity of plane-parallel plates of BK7 (starting from ultra-thin plates and increasing thickness ten times in each consecutive figure; plate thickness is expressed in quarter-wave optical thickness for $\lambda$ = 632.8nm) for coherent radiation of 632.8nm in function of the incidence angles of radiation. Every figure shows the comparison of transmission characteristics of different plates whose optical thicknesses change in one phase thickness range for wave 632.8nm. The following rules can be noticed:

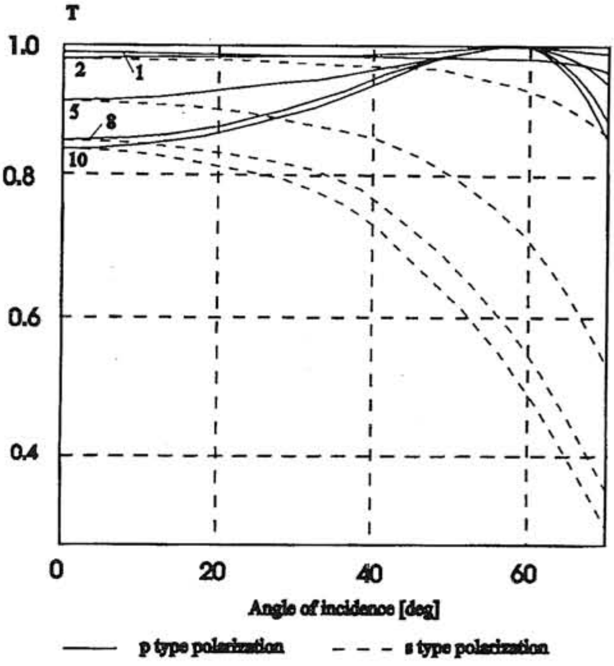

*Fig. 2. Dependence of the transmissivity of a plane parallel plate with BK7 for coherent radiation wavelength 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0–0.0, 2–0.2, 5–0.5, 8–0.8, 10–1.0.*

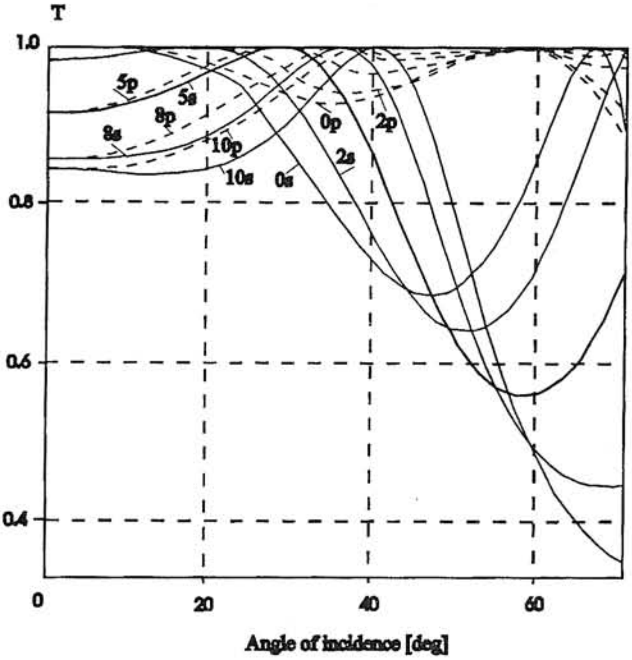

*Fig. 3. Dependence of the transmissivity of a plane parallel plate with BK7 for coherent radiation wavelenght 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0s and 0p – 10.0, 2s and 2p – 10.2, 5s and 5p – 10.5, 8s and 8p – 10.8, 10s and 10p – 11.0; corresponding to s- and p- type polarization.*

1. In the wide range of plate thickness the value of transmission coefficient for normal incidence changes from 84.3% to 100% and it depends only on a fractional number of phase thickness for selected wavelength.

2. With increasing of plate thickness, the interference fringes fall more within their envelopes.

3. For any plate thickness the changes of transmission are slowest for small incidence angles ( in the interval of angles from 0° to 20°).

4. With increasing of the incidence angle, the envelopes of transmissivity for p-type polarization converge and osculate at Brewster's angle. Then they diverge continuously for angles greater than Brewster's angle (Fig.1).

5. With increasing of the incidence angle, the envelopes of changes of transmissivity for s-type polarization diverge to maximum.

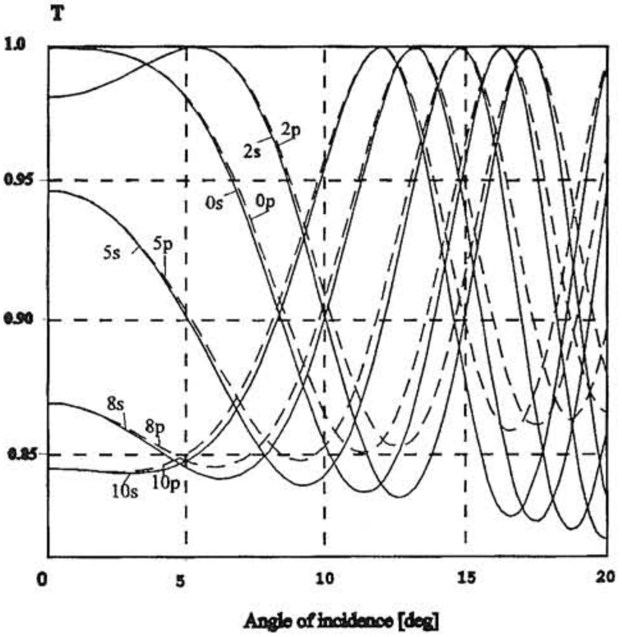

*Fig. 4. Dependence of the transmissivity of a plane parallel plate with BK7 for coherent radiation wavelenght 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0s and 0p – 100.0, 2s and 2p – 100.2, 5s and 5p – 100.5, 8s and 8p – 100.8, 10s and 10p – 101.0; corresponding to s- and p- type polarization.*

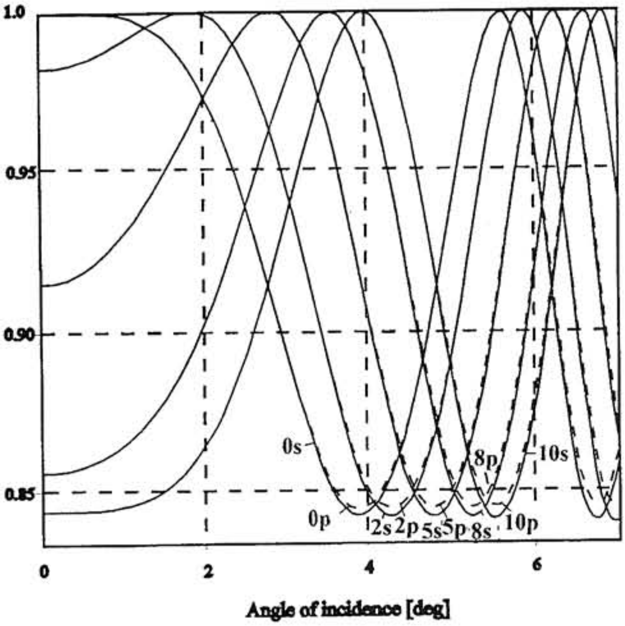

*Fig. 5. Dependence of the transmissivity of a plane parallel plate with BK7 for coherent radiation wavelenght 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0s and 0p – 1000.0, 2s and 2p – 1000.2, 5s and 5p – 1000.5, 8s and 8p – 1000.8, 10s and 10p – 1001.0; corresponding to s- and p- type polarization.*

6. In the case of laser beam radiation (quasi-monochromatic, divergent beam and non-uniform wave front) the transmission can be described as average over spectral and angular energy distribution of the incidence beam. The values achieved for non-coherent radiation are also obtained for averaged values of a coherent radiation passing through thick plates (the average results from spectral and angular distribution of laser beam).

7. It is important to remember that the measured values of transmissivity are averaged (they have statistical importance), but a real instantaneous value can be considerably different.

The transmissivity (in the narrow range of spectrum between 626–636 nm) of BK7 glass plate is presented in Fig.7. The plate thickness equals to 960.8 in quarter-wave optical thickness for 632.8 nm. The coherent radiation falls normally upon the plate.

The best way to check if the plane-parallel plate is suitable, is to measure changes of transmissivity for incidence angles close to 0° (items 1–3). In the case of using a standard He-Ne laser (e.g. LG600 PZO) and applying a computer programme of algorithm based on formula (1), it is possible to determine the thickness of plate with accuracy equal to a decimal part of phase thickness. The accuracy of measurement of the transmitted energy ( items 6 and 7 ) limits the improvement of the precision of thickness's determination. Precision improvement is possible by reducing of divergence of laser beam, e.g. by using additional elements for beam focusing or by using a monomode laser.

This method can be also used for less absorbing materials and for computation of the physical thickness of samples and their complex refraction indices. In these cases the analytical formulas are more complicated.

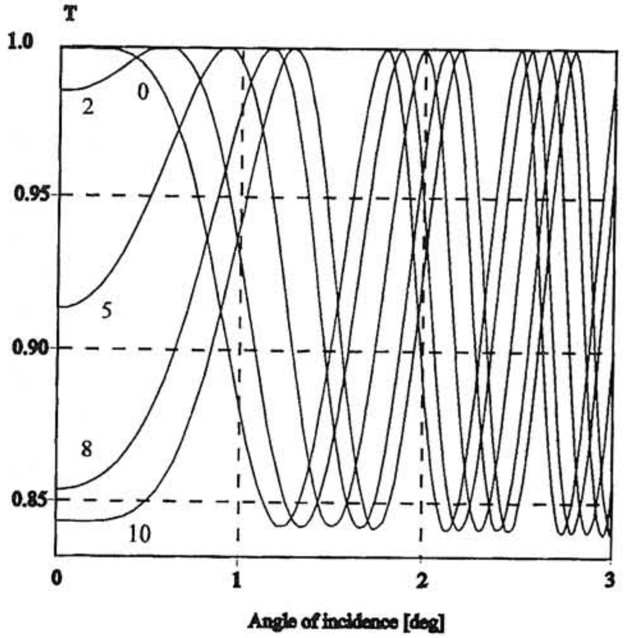

*Fig. 6. Dependence of the transmissivity of a plane parallel plate with BK7 for coherent radiation wavelenght 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0s and 0p – 10000.0, 2s and 2p – 10000.2, 5s and 5p – 10000.5, 8s and 8p – 10000.8, 10s and 10p – 10001.0.*

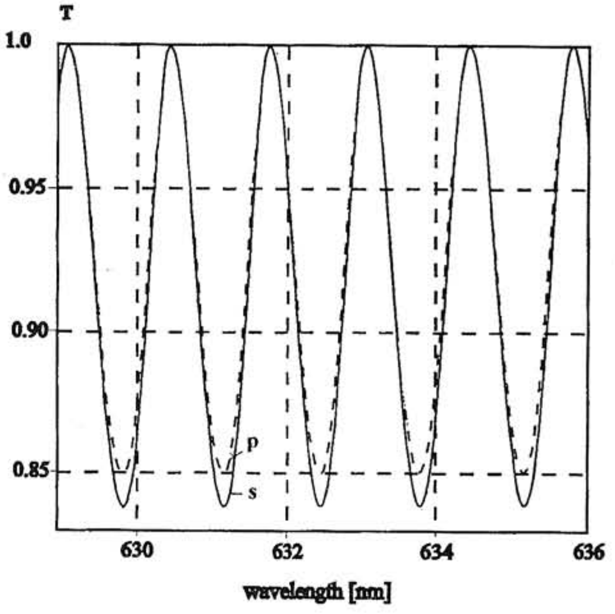

*Fig. 7. Transmissivity of a plane parallel plate with BK7 for coherent radiation incidencing normally. The thickness 960.8 is expressed in quarter-wave optical thickness for 632.8 nm. s and p – corresponding to s- and p- type polarization.*

## 3. Experiment

The measurement of spectral characteristic was performed for over-exposed and chemically fixed photo-foil using a Perkin-Elmer spectrophotometer type Lambda 2. This spectral characteristic is shown in Fig. 8. The photo-foil thickness (d = 115 $\mu$m ± 10 $\mu$m) was measured by means of micrometer screw. The transmissivity for $\lambda$ = 632,8 nm was 77,07%. During the measurement procedure some instabilities were observed, in spite of the fact that external conditions have been stable. This effect was especially distinct in the range of higher wave lengths.

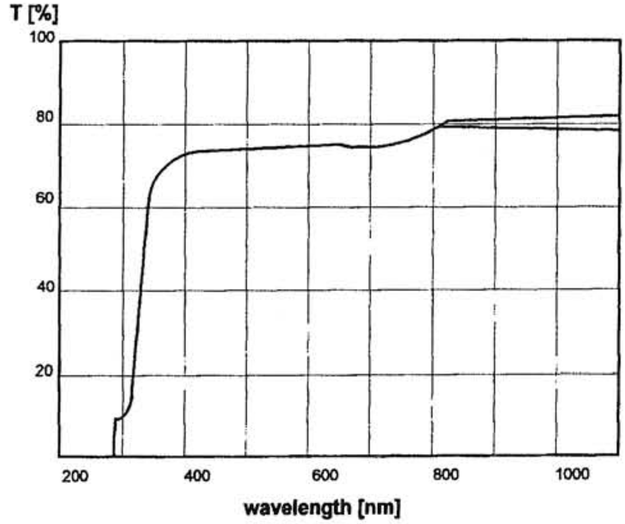

*Fig. 8. The spectral characteristic of the photo-foil at normal incidence.*

The same photo-foil was also investigated in the laser He-Ne system, which generated the beam with a main mode of a power of 3,3 mW. The sample was placed in the special holder, just behind a diaphragm in order to diminish the influence of He-Ne laser beam divergence. Due to this holder the rotation accuracy of 0.1° was obtained. In spite of the application of standard energy meter, the fluctuation of energy indications were in the range: ±2,5% for normal beam incidence, and in the range ±0,6% for angle beam incidence. The results of transmission measurements of the photo-foil in function of an angle of incidence are demonstrated in Fig. 9. From this characteristic the following conclusions can be drawn:

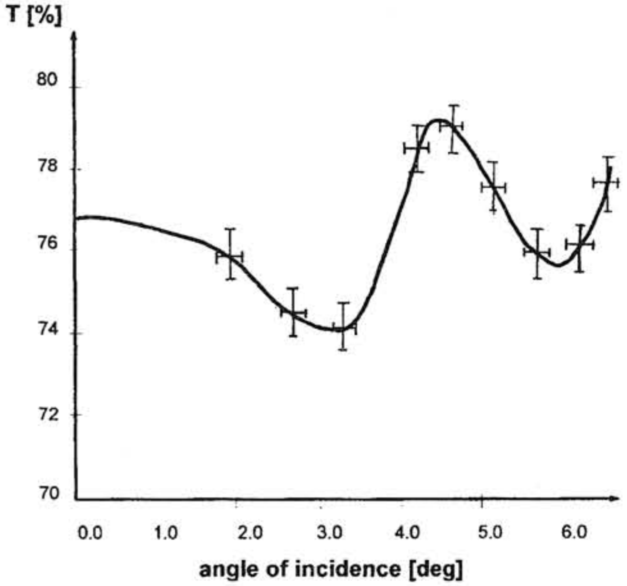

*Fig. 9. Transmission dependence of the photo-foil on incidence angle of 632.8 nm: wavelength radiation of a He-Ne laser.*

1) After the photo-processing the chemical coating is left which results in a partial absorption of laser light. This is the reason of decreasing in the transmitted radiation and also decreasing in the amplitude of signal oscillations.

2) There is a big difference in measuring errors for normal and angle beam incidence. The existing measurement error for normal incidence radiation practically determine the range of transmission oscillations for a small angle of incidence. This is due to the fact, that applied laser, in spite of a generation in main mode, is not a source of single-frequency beam. Thus, the averaging of measurement results takes place for all generated wavelegths. It makes impossible the determination of the layer optical thickness with the accuracy equal to part of phase. Generally in optics phase dependencies determination is of a great importance.

3) For angle beam incidence, the measurements averaging also takes place, but then phase dependencies are not so essential. The foil thickness determined by this method is 122 μm. Not very precise result for normal incident beam enables the determination only of fractional part of phase thickness.

## 4. Conclusions

More detailed data concerning theoretical part of this paper could be found in [7]. The results of measurements of transmissivity dependence on the angle of incidence in the range 0°−10° make possible the satisfactory estimation of influence of a plane-parallel plate on the spectral distribution of the coherent beam. The measurement enables determination of fractional part of phase thickness of plate for applied wavelength. The necessity of initial investigation of the plate with the laser beam becomes obvious. In order to achieve a higher quality of optical elements it is necessary to use more presize measurement equipment. In this case the source of single-frequency beam is needed.

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