Contributed paper

Fourier transform analysis of the spectral modulations obtained at the output of a two-mode optical fiber

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The mutual interference of two linearly polarized (LP) modes is demonstrated in the spectral domain at the output of an optical fiber excited by two low-coherence sources having different spectral widths. The Fourier transform analysis of two measured spectral modulations with the wavelength-dependent periods of modulation affected by intermodal dispersion is successfully performed and a good agreement between the theoretical and the experimental spectral modulations is achieved and the characteristics such as the unmodulated spectra, the wavelength dependences of both the visibility of spectral fringes and the group optical path difference between both LP modes are obtained. These last two characteristics are also compared with those obtained using the cross-correlation technique.

1. Introduction

During the past few years it has been clearly demonstrated that optical interferometry in the spectral domain plays a key role in theoretical and experimental investigations of a large number of optical phenomena. It is being used, for example, to analyse the effects of the state of coherence of light on the spectrum of the light which the sources generate [1, 2] or to analyze optical media via the modulated (channelled) spectrum. The practical importance of the latter case is supported by the fact that the spectral interference of light beams occurs even if the optical path difference between beams appreciably exceeds the source coherence length. Thus, the modulations of spectra of low-coherence sources in a Michelson interferometer has been presented [3, 4] for measuring displacements ranging up to hundreds of micrometres. The real-time interferometric, high-precision measurement of the refractive index of a specimen as a function of the wavelength has been presented [5] from a single interferogram displayed in the spectral domain. White-light interferometry with channelled spectrum detection has been used in optical profilometry [6] and the use of dispersive white-light interferometers for absolute distance measurements, including effects of dielectric multilayer system, has been demonstrated [7, 8]. The spectral interference between two linearly polarized (LP) modes has been used in the evaluation of the group-delay time difference between two LP modes [9, 10].

In this paper we extend the demonstration of the interference experiment presented in previous papers [9, 10] via the spectral modulations obtained at the output of a two-mode optical fiber excited by two low-coherence sources having different spectral widths. Moreover, the Fourier transform method is successfully applied to the analysis of the two measured spectral modulations with the wavelength-dependent periods of modulation affected by intermodal dispersion and good agreement between the theoretical and the experimental spectral modulations is achieved and the characteristics such as the unmodulated spectra, the wavelength dependences of both the visibility of spectral fringes and the group optical path difference (OPD) between both LP modes are obtained. These last two characteristics are also com-

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pared with those obtained using the cross-correlation technique.

2. Experimental configuration

The experimental set-up used in the investigation of the spectral interference between two LP modes of an optical fiber is shown in figure 1. It is the same as that used in the previous work [10]. Two Toshiba TOLD laser diodes (LDs) were used as a source; the first LD

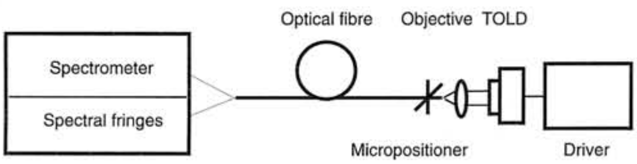

Fig. 1. Experimental set-up for the investigation of the spectral interference between two modes of an optical fiber.

was TOLD 9211 and the second LD was TOLD 9140 operating below the threshold [11] and having a wider spectrum. By using a microscope objective the light from the LD was launched into a step-index optical fiber of length $z = 1.76$ m having a cut-off wavelength for the LP_{11} mode of 920 nm [9, 10]. At the output of the optical fiber, the intensity distribution corresponding to two LP spatial modes, namely the LP_{01} and LP_{11} modes, was resolved. The position of the exit face of the two-mode optical fiber with respect to the entrance slit of the spectrometer, comprising a grating monochromator, was adjusted so that the part of the output optical field with the right lobe of the LP_{11} mode and with the highest field overlap of both LP modes was projected onto the entrance slit of the spectrometer [10].

3. Theoretical background

The mutual interference of two LP modes at an arbitrary point with transverse position vector **r** at the output of an optical fiber of length z is governed at the wavelength by the spectral interference law [1, 9, 10]

$$S(\mathbf{r}, z; \lambda) = S_0(\mathbf{r}; \lambda) + S_1(\mathbf{r}; \lambda) + 2\sqrt{S_0(\boldsymbol{r}; \lambda)\, S_1(\boldsymbol{r}; \lambda)}\cos\left[\frac{2\pi}{\lambda}\Delta_{10}(z; \lambda)\right], \qquad (1)$$

where $S_0(\mathbf{r}; \lambda)$ and $S_1(\mathbf{r}; \lambda)$ are contributions of both modes to the resultant spectral power density $S(\mathbf{r}, z; \lambda)$ and $\Delta_{10}(z; \lambda)$ is the optical path difference (OPD) between both modes, which is expected to be real for a loss-less optical fiber and is related, under the condition of zero phase difference for zero length of the optical fiber, to the phase difference between both LP modes [9, 10]

$$\Delta_{10}(z; \lambda) = \frac{\lambda}{2\pi}\left[\beta_1(\lambda) - \beta_0(\lambda)\right] z\,, \qquad (2)$$

where $\beta_0(\lambda)$ and $\beta_1(\lambda)$ are propagation constants of both modes at a given wavelength λ. Let us consider now that the output optical field of the optical fiber around the transverse coordinate **R** is spectrally analyzed by projecting the light from the optical fiber onto the entrance slit of a spectrometer. Next, we assume that the spectrometer, comprising a grating monochromator, output optics and photodetector, is characterized at the wavelength by the response function $R(\lambda)$. Spectral power or spectrum at the output of the spectrometer associated with the transverse coordinate **R** can be expressed by the well-known convolution relation [10]

$$I(\mathbf{R}, z; \lambda) = \int_A \int_\Lambda S(\mathbf{r} + \mathbf{R}, z; \lambda')R(\lambda - \lambda')\mathrm{d}^2\mathbf{r}\mathrm{d}\lambda'\,, \qquad (3)$$

where the spatial integration is performed over the area A of the entrance slit of the spectrometer and the wavelength integration is performed over the wavelength range Λ much larger than the width of the response function $R(\lambda)$. On substituting equation (1) into equation (3), we obtain

$$I(\mathbf{R}, z; \lambda) = I_0(\mathbf{R}; \lambda)\left\{1 + V_1(\boldsymbol{R}, z; \lambda)\cos\left[\frac{2\pi}{\lambda}\Delta_{10}(z; \lambda)\right]\right\}, \qquad (4)$$

where $I_0(\mathbf{R}; \lambda)$ is the reference spectral power in which the effect of both the source spectrum and the response function of spectrometer is inscribed and $V_I(\mathbf{R}, z; \lambda)$ is the visibility of the spectral fringes in which the effect of excitation conditions of both modes, the effect of the period of the spectral modulation dependent on the optical fiber length z as well as the effect of the response function of the spectrometer is included. One may conclude from equation (4) that the spectral modulation with the period of modulation

$$\Delta\lambda_{10}(z; \lambda) = -\frac{\lambda^2}{\Delta^{g}_{10}(z; \lambda)}\,, \qquad (5)$$

is to be expected whenever the spectral analysis is performed with a spectrometer of a suitable resolution.

In equation (5), $\Delta^g_{10}(z;\,\lambda)$ is the group OPD between both modes given by

$$\Delta^g_{10}(z;\lambda) = \Delta_{10}(z;\lambda) - \lambda\frac{d\Delta_{10}(z;\lambda)}{d\lambda}. \qquad (6)$$

Using the Fourier transform or the cross-correlation method for the spectral fringe analysis, the wavelength dependence of both the group OPD between modes and the visibility of the spectral fringes can be extracted from the measured spectral modulation.

3.1 Fourier transform method of fringe pattern analysis

The spectral modulations, that is spectral interferograms, have a general form (4) which can be related to the similar form of one-dimensional spatial interferograms (the spatial coordinate is replaced by the wavelength) [12-14]

$$g(\lambda) = a(\lambda) + b(\lambda)\cos\left[2\pi f_0\lambda + \phi(\lambda)\right], \qquad (7)$$

where $a(\lambda)$ and $b(\lambda)$ represent background and modulation terms respectively; f_0 is the carrier frequency in the wavelength domain and $\phi(\lambda)$ is the phase function. In practical applications [4], $a(\lambda)$, $b(\lambda)$ and $\phi(\lambda)$ are slowly varying functions compared to the variations introduced by the carrier frequency f_0. Using the Fourier transform method for the spectral fringe analysis, which was originally conceived and demonstrated by Takeda *et al.* [12] for extracting phase information from the fringe pattern arising out of the interference of tilted wavefronts, the terms $a(\lambda)$ and $b(\lambda)$, the carrier frequency f_0 and the phase function $\phi(\lambda)$, which has to be corrected by using a phase-unwrapping algorithm [12-14], can be obtained. Correspondingly, from a comparison of equation (4) and equation (7) results that the reference spectrum $I_0(\mathbf{R};\lambda)$, the visibility of the spectral fringes $V_I(\mathbf{R}, z; \lambda)$ and the overall phase $\Phi(z;\lambda) = \Delta_{10}(z;\lambda)2\pi/\lambda = 2\pi f_0\lambda + \phi(\lambda)$ can be obtained as we have demonstrated in the most recent work [4] in which three spectral interferograms measured at the output of the uncompensated (dispersive) Michelson interferometer have been processed.

3.2 Cross-correlation method

The wavelength dependence of both the visibility of the spectral interference fringes and the group OPD between modes can be obtained using the cross-correlation procedure. It is based on the fact that both these parameters are over a spectral region, in which the cross-correlation method of processing the measured spectral modulations is performed, independent of the wavelength. If the spectrum $I(x;\lambda_n)$ is measured at discrete wavelengths λ_n, where $n = 1, 2, ..., N$ and N is a number of points over a finite spectral interval, a normalized cross-correlation function $\gamma(x)$ between the measured spectrum $I(x;\lambda_n)$ and the sampled theoretical spectrum $I_T(x;\lambda_n)$ is defined as

$$\gamma(x) = \frac{\sum_{n=1}^{N} I_T(x,\lambda_n)\, I(x,\lambda_n)}{\sqrt{\sum_{n=1}^{N} I_T^2(x,\lambda_n)}\,\sqrt{\sum_{n=1}^{N} I^2(x,\lambda_n)}} \qquad (8)$$

where x is a quantity representing the visibility of the spectral interference fringes or the group OPD between modes. An estimate for the quantity x within a given spectral region is determined as being that value of x for which $\gamma(x)$ is maximized.

4. Experimental results and discussion

In order to demonstrate the applicability of both the Fourier transform method in the spectral fringe analysis and the cross-correlation procedure for obtaining the group OPD from the measured spectral modulations, we use first of all the measured normalized spectrum mentioned in the previous paper [10]. Figure 2 shows by markers the normalized spectrum measured at the output of a two-mode optical fiber excited by the TOLD 9211. The spectrum was measured in the range of wavelengths from 669 to 674

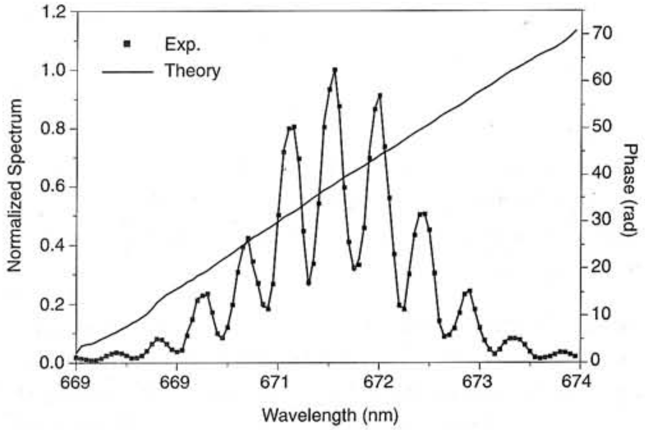

Fig. 2. Measured normalized spectral modulation for TOLD 9211 and results of its processing: theoretical spectrum and unwrapped overall phase (bold curve).

nm with a 0.05 nm step between two measurements by the spectrometer whose width of the entrance slit was selected to be 50 μm. In figure 2 is also shown by a full curve the corresponding theoretical normalized spectrum resulting from the spectral modulation given by equation (4). The theoretical spectral modulation is obtained by processing the measured spectral modulations using the Fourier transform method for the spectral fringe analysis. We see good agreement between the theory and the experiment. By processing the measured spectral modulation and using a phase-unwrapping algorithm the wavelength dependence of the overall phase $\Phi(z; \lambda)$ is obtained. It is shown in figure 2 by a bold curve. This curve is non-linear. This means that it is affected by intermodal dispersion.

The wavelength dependence of the group OPD between both modes can be evaluated using equation (6) in which the OPD $\Delta_{10}(z; \lambda)$ is related to the overall phase $\Phi(z; \lambda)$ through a relation $\Phi(z; \lambda) = \Delta_{10}(z; \lambda)2\pi/\lambda$. Fortunately, even if the phase $\Phi(z; \lambda)$ and correspondingly the OPD $\Delta_{10}(z; \lambda)$ is known with ambiguity, it results from equation (6) that the group OPD is not affected by it. The wavelength dependence of the absolute value of the group OPD obtained from the measured spectral interferogram shown in figure 2 is depicted in figure 3. Despite the errors near edges of

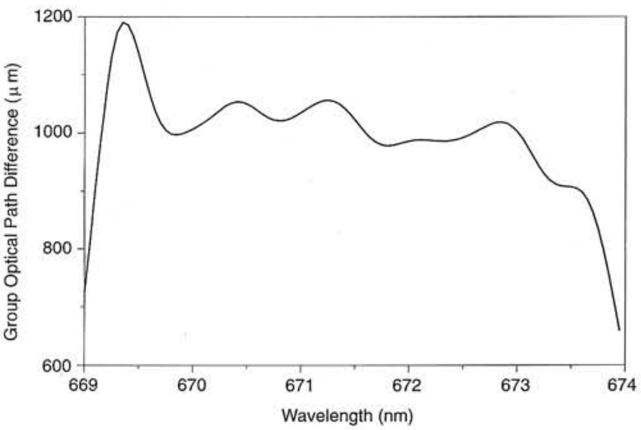

Fig. 3. The wavelength dependence of the absolute value of the group optical path difference between two modes obtained by processing the measured spectral modulation shown in figure 2.

the measured spectral region, it slightly decreases with increasing wavelength. If we assume that the group OPD between modes is the wavelength-independent, the cross-correlation technique can be used. In figure 4, the degree of the correlation between the measured and the theoretical spectral modulations (the normalized cross-correlation function) is shown as a function of

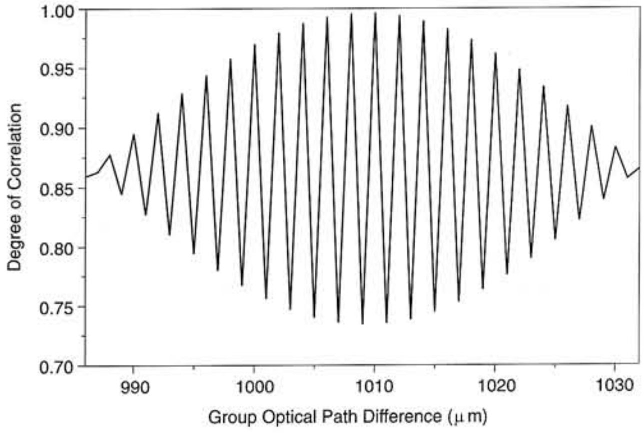

Fig. 4. Degree of correlation between the measured and the theoretical spectral modulations as a function of the group optical path difference between modes.

the group OPD. It is clearly seen that this dependence is maximized for the group OPD between both modes of 1010 μm. This value is in good agreement with that in figure 3 corresponding to the source central wavelength.

If we want the intermodal dispersion to be more pronounced, we use the TOLD 9140 with a wider spectrum. Spectral modulation measured at the output of the two-mode optical fiber in the range of wavelengths from 675 to 705 nm with a 0.05 nm step between two measurements by the spectrometer whose width of the entrance slit was selected to be 100 μm is shown in figure 5. By processing this measured spectral modulation using the Fourier transform method for the spectral fringe analysis, the unmodulated spectrum $I_0(\mathbf{R}; \lambda)$ depicted in the same figure 5 was obtained. It can be seen that a central wavelength λ_0 of 690.8 nm as well as the full width at half-maxi-

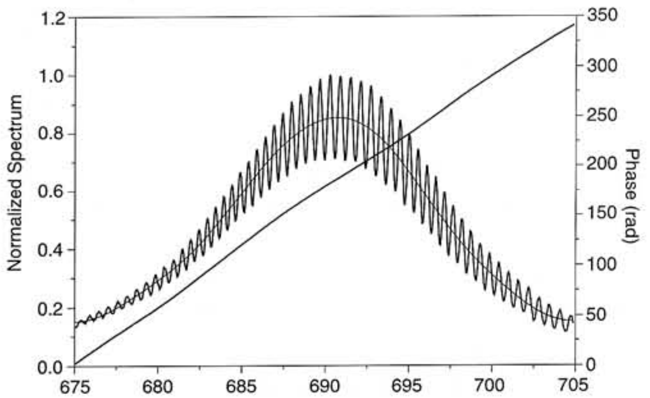

Fig. 5. Measured normalized spectral modulation for TOLD 9410 and results of its processing: unmodulated spectrum and unwrapped overall phase (bold curve).

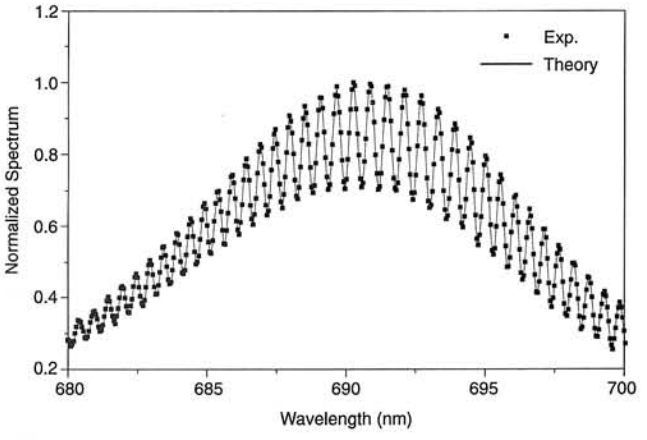

Fig. 6. Comparison of the measured normalized spectral modulation with the theoretical spectral modulation within a narrower spectral region.

mum $\Delta\lambda$ of 15.4 nm can be attributed to the spectrum of the LD. The corresponding coherence length $l_c \approx \lambda_0^2/\Delta\lambda$ is 30 μm. In the same figure 5 is also shown the overall phase $\Phi(z;\ \lambda)$. The efficiency of the above-presented procedure can be clearly demonstrated by a comparison of the measured and the theoretical spectral modulation. Both spectral modulations are shown within a narrower spectral region in figure 6 from which we see very good agreement.

Processing the measured spectral modulation shown in figure 5 using the Fourier transform method for the spectral fringe analysis also gives the most valuable information regarding both the visibility of the spectral fringes $V_I(\mathbf{R},\ z;\ \lambda)$ and the group OPD $\Delta^g{}_{10}(z;\ \lambda)$ between both LP modes. The wavelength dependence of the visibility of the spectral fringes is shown in figure 7. We can conclude that the larger the wavelength, the higher is the visibility. As can be seen from equations (1) and (4) this effect can mainly be attributed to the spectrally dependent overlap of both LP modes. Figure 7 also demonstrates errors in the visibilities near both edges of the spectral region which are owing to the phase evaluation errors [14]. The wavelength dependence of the visibility of spectral fringes can also be obtained by using the cross-correlation procedure. The cross-correlation between the measured and the theoretical spectral modulation was performed in the narrower spectral regions, within which the visibilities were considered to be spectrally independent. Repeating this procedure over the whole measured spectral region the corresponding wavelength dependence was obtained. Figure 7 clearly demonstrates that the results obtained by application of both techniques agree well.

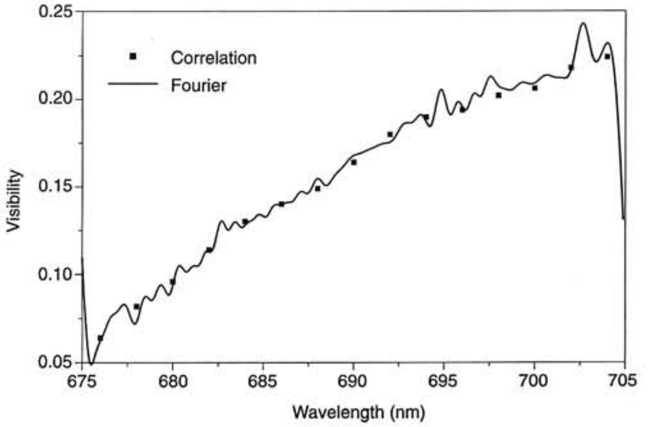

Fig. 7. The wavelength dependence of the visibility of spectral interference fringes obtained by processing the measured spectral modulation shown in figure 5.

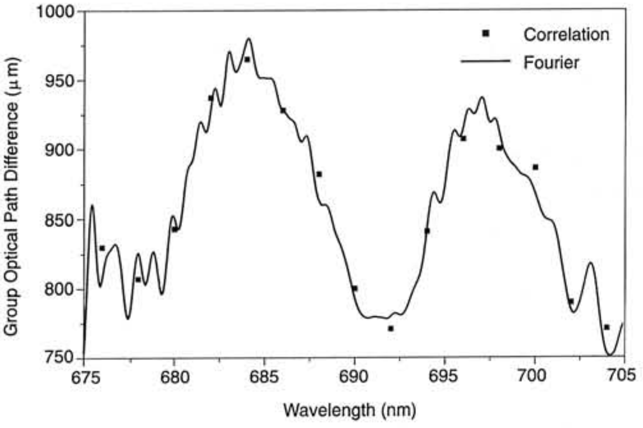

Fig. 8. The wavelength dependence of the absolute value of the group optical path difference between two modes obtained by processing the measured spectral modulation shown in figure 5.

The wavelength dependence of the absolute value of the group OPD between both LP modes is shown in figure 8. It can be seen from figure 8 that the wavelength dependence of the group OPD includes two clearly resolved maxima. Repeating the spectral measurements at the output of the two-mode optical fiber the reproducibility of this effect was confirmed. We should note that the wavelength dependences of both the visibility of spectral fringes and the group OPD between two LP modes can be compared with those resulting from the corresponding theory, including equations (1), (2), (4)-(6), but this aspect is not considered in this paper. The wavelength dependences of the group OPD can also be obtained by using the cross-correlation procedure. The cross-correlation between the measured and the theoretical spectral modulation was performed in the narrower spectral regions, within which the group OPDs were considered to be spectrally independent. Repeating this procedure over the whole measured spectral region the corresponding wavelength dependence was obtained. Figure 8 clearly demonstrates that the results obtained by application of both techniques are in good agreement.

5. Conclusions

We have demonstrated that spectral interferometry can be used efficiently to measure in situ the effects of modal fields and intermodal dispersion on the spectral modulations obtained at the output of a two-mode optical fiber. Consequently, we have demonstrated that the spectral modulations can be successfully processed using the Fourier transform method for the spectral fringe analysis to obtain the wavelength dependences of both the visibility of spectral fringes and the group OPD between both modes. We have shown that these two characteristics can be also obtained using the cross-correlation technique; good agreement between the results of both techniques was achieved. All obtained spectral characteristics of a fiber waveguide can be compared with results of an adequate theoretical model. The above-presented method based on low-coherence interferometry can be readily included in the measurement techniques for the dispersion characterization of optical fibers.

References

1. L. Mandel, E. Wolf: *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge 1995).
2. E. Wolf, D.F.V. James, Rep. Prog. Phys., **59** (1996) 771.
3. P. Hlubina, J. Mod. Optics, **44** (1997) 1049.
4. P. Hlubina, Optoelectronics Review **5** (1997) 117.
5. C. Sáinz, P. Jourdain, R. Escalona, J. Calatroni, Optics Commun., **110** (1994) 381.
6. J. Schwider, L. Zhou, Opt. Lett., **19** (1994) 995.
7. U. Schnell, E. Zimmermann, R. D ndliker, Pure Appl. Opt., **4** (1995) 643.
8. U. Schnell, R. D ndliker, S. Gray, Opt. Lett., **21** (1996) 528.
9. P. Hlubina, J. Mod. Optics, **43** (1996) 1745.
10. P. Hlubina, Opt. Engng., **36** (1997) 831.
11. D. A. Christensen, J. Rotgé, A. Klemas, G. Loos, D. Merriman, Opt. Engng., **33** (1994) 2034.
12. M. Takeda, H. Ina, K. Kobayashi, J. Opt. Soc. Amer., **72** (1982) 156.
13. M. Takeda, Ind. Metrology, **1** (1990) 79.
14. R.J. Green, J.G. Walker, D.W. Robinson, Opt. Lasers Eng., **8** (1988) 29.