New polarimetric detection system

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An idea of a new polarimetric detection system is presented. This system can be used to measure parameters of polarization ellipse. The system controlled by computer consists of a tunable retarder, analyzer and detector. The change of detector signal related to the change of retardation on a liquid-crystalline retarder allows to obtain parameters of polarization ellipse. Theoretical considerations have been confirmed by an experiment.

1. Introduction

The state of polarization (SOP) is one of the main parameters of light. If SOP is constant in time, then a detection system consisting of a rotating quarter-plate placed before the detector and a polarizer is usually used to measure this state. An analysis of SOP is performed by a comparison of intensity of incident light at the detector for different values of azimuth of the quarter-plate and the polarizer [1]. Another approach is used in case of the measurements of SOP which changes in time. In the simplest case (when only rotation of light polarization plane exists) the measured beam is divided into two paths with orthogonal linear polarization states. Then intensity of beam in those two paths is measured [2]. However, if enforced linear birefringence emerges, the resulting change of intensity on detectors will be interpreted as an additional twist of a polarization plane of light. For a measurement of any SOP the measured beam is divided into two paths having different polarization properties [3, 4]. By the measurement of variable light intensity in both paths one can obtain the change of parameters of polarization ellipse in time. The essential problem in this case is to obtain defined polarization properties in each path and stabilization of these properties against action of external factors.

The idea of the polarimetric detection system presented in this paper is a new approach to the problem of SOP measurements. Measured polarization at chosen moment is an averaged value for one period of driving signal on the retarder. In case of tunable liquid crystal retarder [5], this period has an order of tenth parts of a second. Presented method can be used for measurements of SOP varying in time.

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2. The idea of the detection system

The scheme of applied detection system is presented in Fig. 1. This system consists of a tunable retarder (TR), a polarizer (P), a detector (D), and a personal computer (PC). For suitably chosen azimuths of the retarder and polarizer, one can find full information about SOP of light beam.

The retardation between orthogonal polarizations of light changes in accordance with retardation characteristic of TR [6].

To find the dependence between the detector signal and given SOP one can discuss the detection system of a polarimeter, presented in Fig. 1, in terms of Jones' matrix calculus [7]. Vector of electric field before detection system is described by the so called standard Jones' vector E_{in} expressed as follows:

$$E_{in} = \begin{bmatrix} \cos\phi \\ \sin\phi \cdot e^{j\Delta} \end{bmatrix} \quad (1)$$

where ϕ and Δ stand for chosen parameters of polarization ellipse, namely diagonal angle and retardation of orthogonal components of electric field, respectively. These parameters change in the following ranges:

$$0 \le \phi \le \pi/2, \quad 0 \le \Delta \le 2\pi \quad (2)$$

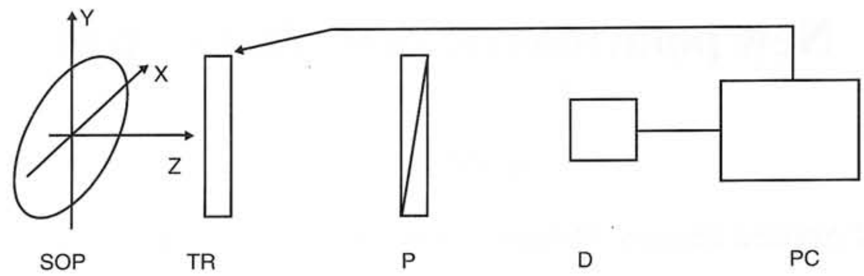

Fig. 1. The scheme of the detection system of a polarimeter.

TR is adjusted in such a way that its fast axis is parallel to the X axis as it is shown in Fig. 1. Mathematically TR is described by Jones' matrix of linear retarder G(δ), where δ describes the retardation between X and Y components and it depends on used voltage:

$$G(\delta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\delta(V)} \end{bmatrix} \tag{3}$$

The next element of the system is a polarizer oriented at an azimuth β with respect to the X axis. The Jones matrix of this element can be expressed as follows:

$$E_{in} = \begin{bmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{bmatrix} \tag{4}$$

Electric field before the detector is described by the following dependence:

$$E_{out} = P(\beta)\cdot G(\delta)\cdot E_{in} = \begin{bmatrix} \cos\phi\cos^2\beta + \sin\phi\sin\beta\cos\beta e^{j(\Delta-\delta)} \\ \cos\phi\sin\beta\cos\beta + \sin\phi\sin^2\beta e^{j(\Delta-\delta)} \end{bmatrix} \tag{5}$$

Normalized intensity of light incident at the detector is described by the formula:

$$I = \cos^2\phi\cos^2\beta + \sin^2\phi\sin^2\beta + \frac{1}{2}\sin 2\phi\sin 2\beta\cos(\Delta-\delta) \tag{6}$$

The analysis of Eq. 6 shows that it is useful to adjust the azimuth β = π/4 or β = – π/4 to obtain the highest sensitivity. In this case normalized intensity is described as:

$$I(V) = \frac{1}{2}\,[\,1 \pm \sin(2\phi)\cdot\cos(\delta(V)-\Delta)\,] \tag{7}$$

Detected voltage for linear range of work can be described by the following formula:

$$U(V) = U_0\,[\,1 \pm \sin(2\phi)\cdot\cos(\delta(V)-\Delta)\,] \tag{8}$$

where:

$$U_0 = 0.5\,[U(V_1) + U(V_2)] \tag{9}$$

V_1 and V_2 stand for voltages for which two following extrema of U(V) function are observed.

Signs "+" in Eqs. (7-8) describe β = π/4 azimuth, while signs "–" describe β = -π/4 azimuth. Note, that if diagonal angle is equal to zero or π/2 (the one of main axes of TR belongs to plane of input polarization), then $U(V) = U_0$ and polarization is linear.

In general case dividing of the range of diagonal angle measurement to sub-ranges, namely $0 \le \phi \le \pi/4$ or $\pi/4 \le \phi \le \pi/2$ is useful to univocal description of the sign of phase shift and the direction of rotation of the polarization plane. The above statement can be derived from Eq. 8, because for each value of sin(2ϕ) two values of diagonal angle can be attained in the range of variation of this angle. However, usually rotation of a polarization plane is far below the discussed limitation.

3. Mutual measurement of the diagonal angle and retardation

The characteristic of the tunable retarder, i.e. δ(V) function, is known. Let us consider the following range of variation: $0 \le \phi \le \pi/4$. It can be concluded from Eq. 8 that amplitude of changes of the detector signal is $U_0\sin(2\phi)$, while U(V) characteristic shifts along ordinate axis, depending on retardation Δ. The last parameter is controlled by the value of voltage applied to TR. For applied liquid crystal TR, retardation δ(V) is the lowest for high voltages and increases for decreasing voltage. Let us denote voltages for which the first and the second extremum of the detector signal occur (from the side of high voltages) as V_1 and V_2, respectively. Then voltages at the detector in extremum points are given by:

$$U(V_1) = U_0(1 \pm \sin 2(\phi)), \qquad (10\text{-}11)$$
$$U(V_2) = U_0(1 \pm \sin 2(\phi))$$

The positive sign in Eq. 10 and negative one in Eq. 11 exist when the extremum is a maximum, i.e.:

$$|\delta(V_1) - \Delta| = 2\pi n, \qquad (12\text{-}13)$$
$$|\delta(V_2) - \Delta| = (2n + 1)\pi$$

In the first case, when a minimum occurs from the side of higher voltages, one can obtain the following expressions:

$$|\delta(V_2) - \Delta| = (2n - 1)\pi, \qquad (14\text{-}15)$$
$$|\delta(V_2) - \Delta| = 2\pi n$$

where n stands for an integer.

Hence using Eq. (12 -15) one can find the retardation. If a maximum goes before minimum during sliding from high voltage side, the retardation can be calculated from the following formula:

$$\Delta = \delta(V_1) \pm 2\pi n = \delta(V_2) \pm (2n + 1)\pi \qquad (16)$$

If a minimum occurs first, then

$$\Delta = \delta(V_1) \pm (2n - 1)\pi = \delta(V_2) \pm 2\pi n \qquad (17)$$

Note, that from Eqs. 10 and 11 diagonal angle can be directly obtained:

$$\sin(2\phi) = \frac{U_{\max} - U_{\min}}{U_{\max} + U_{\min}} \qquad (18)$$

where U_{max} and U_{min} stand for the highest and the lowest value of the U(V) signal, respectively.

If the diagonal angle differs from zero, the detector signal can reach $U(V) = U_0$ value for:

$$\Delta = \delta(V_3) \pm \frac{\pi}{2} \qquad (19)$$

To obtain the diagonal angle from the whole signal U(V) one can re-write Eq.8 in the following form:

$$U(x) = ax + b \qquad (20)$$

where:

$$a = \pm U_0 \sin 2\phi, \qquad (21\text{-}23)$$
$$b = U_0,$$
$$x = \cos(\delta(V) - \Delta) = \cos\xi(V)$$

$$\cos\xi(V) = \pm\frac{2U(V) - U_{\max} - U_{\min}}{U_{\max} - U_{\min}} \qquad (24)$$

Sign "+" in Eqs. 21 and 24 exists for $\beta = \pi/4$, while sign "–" for $b = -\pi/4$. Parameters a and b are obtained by least squares method and then, using Eq. 21, the diagonal angle ϕ is obtained. Using Eq. 16 or 17 one can also find phase shift Δ.

4. Experimental results

The scheme presented in Fig. 2 shows the measurement system. Light from the source (S) (semiconductor laser with wavelength of 670 nm) is directed for a system used to form the polarization state. The system consists of a polarizer (P1), half-wave plate ($\lambda/2$) and voltage-controlled retardation plate (TR1), which allow to form fixed polarization states.

The detection system, as in Fig. 2, consists of a tunable retarder (TR2), polarizer (P2) adjusted with an azimuth $\pi/4$ rad with respect to the fast axis of TR2, detector (D) and personal computer (PC). As a tunable retarder liquid crystal voltage controlled retardation plate has been applied. The dependence of retardation between orthogonal eigen waves on voltage applied on TR2 has been found experimentally. For this measurement the method described in [6] has been used. Obtained characteristic is presented in Fig. 3.

As one can see, the applied retarder is characterized by a threshold voltage, above which decrease of retardation corresponds to voltage increase. The highest changes of retardation have been obtained for driving voltages from 1.5 to 4 V (see Fig. 3).

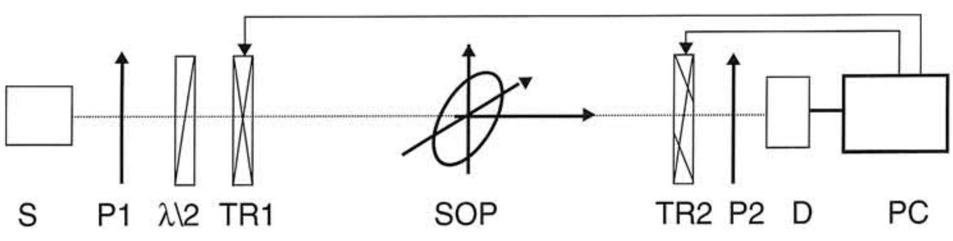

Fig. 2. The scheme of the measurement system.

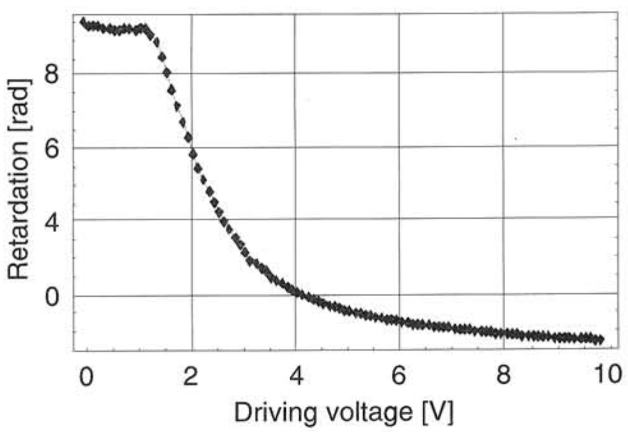

Fig. 3. The dependence of retardation d(V) on amplitude of driving voltage.

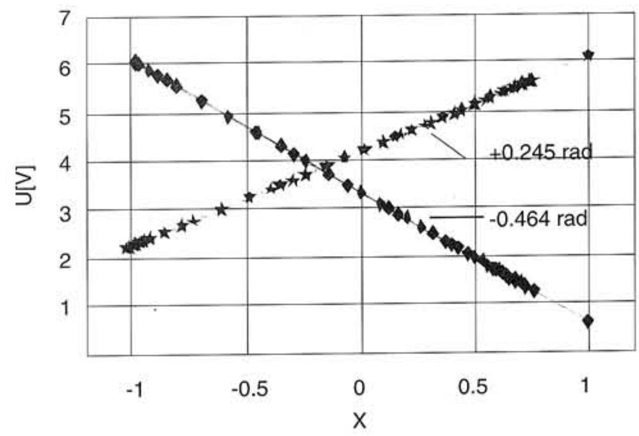

Fig. 5. The dependence U(x) according to Eq. 21 for the same azimuths of polarization as in Fig. 4.

The simplest case is an occurrence of a rotation of the polarization plane only. The dependence of the detector signal and driving voltage for two examples of linear polarization states is given in Fig. 4. It is seen that extrema of the characteristics are placed in the same value of voltage independently on the azimuth of SOP. Moreover an amplitude of signal changes depends on this azimuth.

The azimuth has been obtained by means of the least squares method using Eq. 21 for a change of a driving voltage from 0.0 to 10.0 Volts. The U(x) dependence is illustrated in Fig. 5. As one can see, measurement points of each signal are placed according to theoretical predictions. Standard deviation for rotations of the polarization plane calculated by the above method is less than 0.002 rad.

The presented detection system can be applied for analysing of elliptically polarised light, too.

The examples of the dependences of a detector signal on supplying voltage for chosen adjustment of an azimuth of half-wave plate and different ellipticity of light incident at the detector (which have been obtained by changing of voltage at supplying TR1) are presented in Fig. 6. As a comparison, the dependence of the detector signal on supplying voltage for a certain azimuth of linear polarization behind half-wave plate (system without TR1, $\alpha_1 = 0.393$ rad) is also given in the same figure.

This dependence is illustrated by curve (1). Curves (2) and (3) illustrate dependences of the detector signal on driving voltage for the same azimuth of half-wave plate, but two different voltages supplying TR1. In other words, the SOPs of incident light at the detection system are different. Note, that extrema of curves (2) and (3) occur for driving voltage different than this one for a curve (1). This is the effect of different retardation Δ. For curve (1) the highest value of the detector signal corresponding to retardation by a multiplicity of 2π occurs for V = 2940 mV, while for curves (2) and (3) for V = 2680 mV and V= 2200 mV, respectively. In

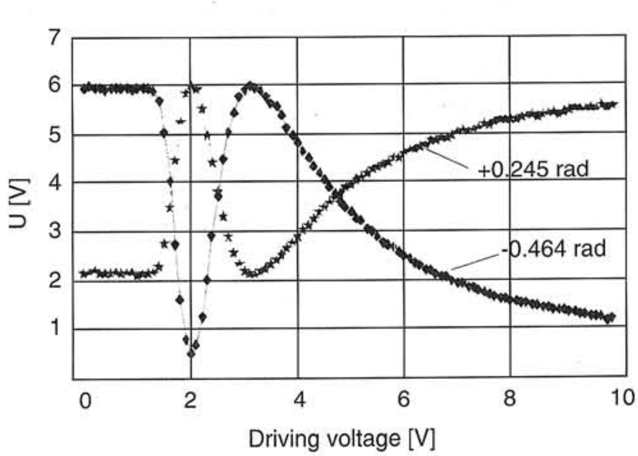

Fig. 4. The dependence between the detector signal and driving voltage (TR2) for two azimuths of input polarization (V1 = const, V2 = const).

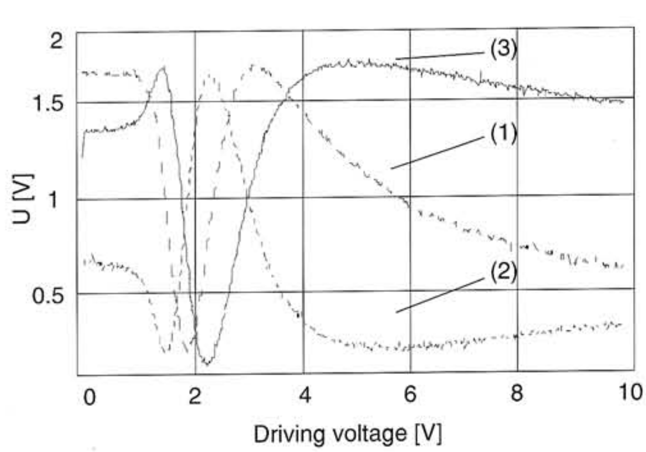

Fig. 6. The dependence of the detector signal on driving voltage for linear polarization - curve (1) and elliptical polarizations - curves (2) and (3).

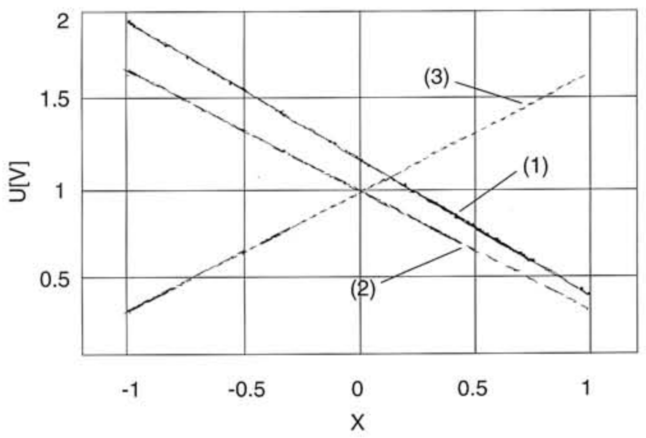

Fig. 7. Curves (1), (2) and (3) represent U(X) characteristics, where $X=\cos(\delta-\Delta)$ for SOP corresponding to Fig. 6.

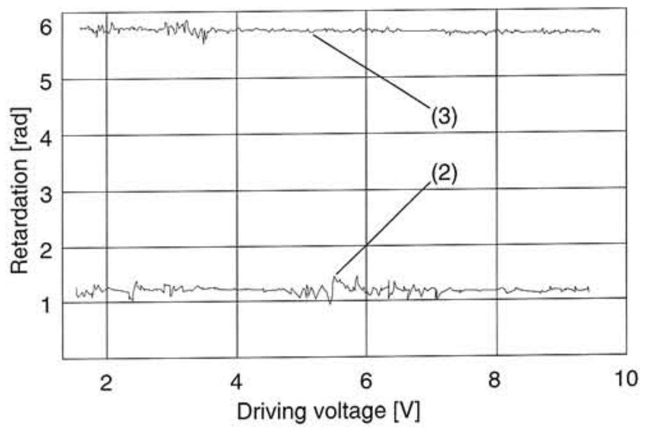

Fig. 9. Retardation Δ for polarization states presented as curves (2) and (3) vs. used driving voltage.

case of curve (3) from the side of higher voltages the first minimum occurres for V = 4100 mV, which means that retardation is larger than π rad.

After application of the least squares method the dependences of the detector signal $U(\cos(\delta-\Delta))$ have been obtained in form presented in Fig. 7.

The measurement points lie on straight lines, similarly as in the case of studies on linear polarization. Using direction coefficients, calculated by the least squares method, the diagonal angles have been obtained, namely $\phi_2 = 0.401$ rad and $\phi_3 = 0.406$ rad. The standard deviation has been less than 0.004 rad.

According to predictions (see Fig. 6) retardation for curve (3) in comparison to (1) is larger than π. Changes of detector voltage for (1) and (3) are opposite. Taking above into account one can notice that polarization state (2) exhibits retardation in the range of $0 < \Delta < \pi$, while state (3) in the range of $0 < \Delta < 2\pi$. The dependence of $(\delta - \Delta)$ on driving voltage is presented in Fig. 8.

Using phase characteristics connected with studied polarisation state and retardation characteristics TR2

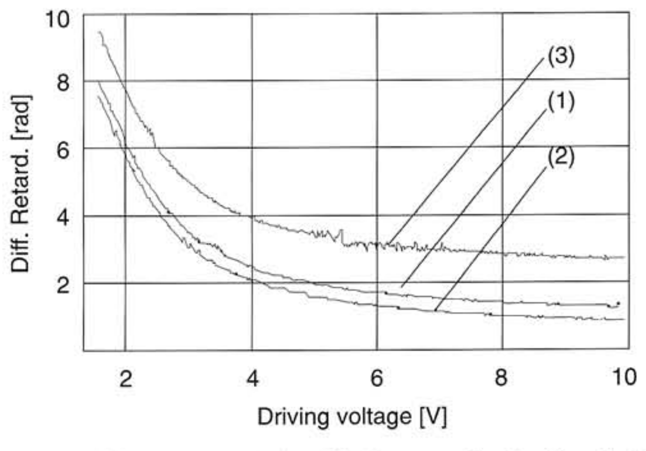

Fig. 8. Difference retardation $(\delta-\Delta)$ v.s amplitude of studied polarization states.

one can obtain the value of retardation for each value of driving voltage, what is illustrated in Fig. 9. Values of retardation obtained by the least squares method have been $\Delta_2 = 0.4563$ rad and $\Delta_3 = 5.053$ rad, respectively. Standard deviations of measurements have been less than 0.004 rad.

5. Summary and conclusions

From conducted theoretical and experimental studies one can conclude that the presented detection system can be successfully adopted for determination of polarization state of light.

In the theoretical description of the SOP the normalised Jones' vector has been used. The diagonal angle and retardation describe SOP completely. The essential element of the detection system is an electrically driven retarder. For the applied range of voltage it gives retardation which allows to measure the following extrema of the detector signal.

The discussed detection system allows to perform an analysis of the SOP without changing azimuths of optical elements in the system. According to results of theoretical analysis, the change of diagonal angle of studied SOP can be observed as a change of extrema of the detector signal, while shift of phase-voltage characteristics corresponds to the change of retardation.

The detection system can be applied in polarimeter systems, moreover, in polarization interferometry. In particular, fibre-optic in-line version can be constructed.

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