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Analysis of working conditions of multimode fibre amplifiers

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The paper presents theoretical background of radiation amplifier for different active optic fibres: multimode and singlemode ones operating in three- or four-level pumping systems and with various doping. The authors did the comparison of working conditions of singlemode and multimode amplifiers. Because of high-level energy losses in the tested multimode fibre also the temperature and noise problems were discussed.

Keywords: optic amplifiers, multimode fibres, power loss.

1. Introduction

A big problem in application of laser distance meters is necessity to use reflector at a measured point or measured range is quite small. There are two possibilities to increae measured range; one is increase in source power, another is input signal amplification. For the latter one multimode fibre amplifiers can be used. The authors of this work made a review on working conditions of multimode fibre amplifiers.

A lot of papers describe singlemode erbium doped fibre amplifiers [1,2] and there are only a few papers on multimode amplifiers and amplifiers based on active dopants different from erbium [3]. The known simplified analysis of singlemode erbium amplifiers [4], based on energy quantum equations, and light propagation equations were generalised by the authors.

2. Theoretical background of amplification in active media

Optic fibre amplifier is important type of amplifiers. A medium amplifying the signal is rare-earths doped and pumped fibre. Optical pumping depends on applied dopants. It can be described by a four-level or three-level energy model [5]. For neodymium a four-level scheme, presented in Fig. 1, is used.

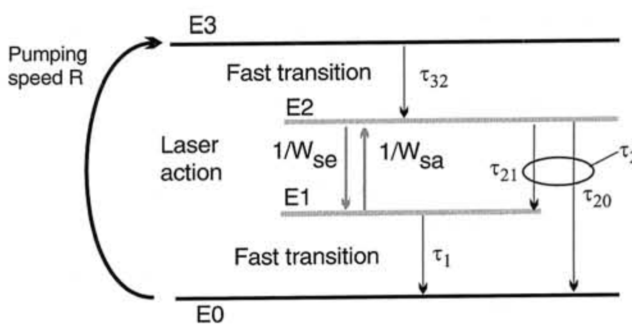

Fig. 1. A diagram of four-level optical pumping for Nd doped glass.

The principle of four-level optical pumping can be described as follows: electrons in the atoms of the dopant are pumped from the basic level E0, by an external source, to level E3. A lifetime of the electrons on level E3 is short and they quickly go on metastable level E2 (the lifetime for Nd equals 0.3 ms). As a result the electrons are gathered on level E2. It enables laser action that is stimulated transition from level E2 to E1. The lifetime of the electrons on level E1 level is short, so all the electrons being on this level quickly change into ground-state level from where they can be pumped again into E3 level. For erbium doped optic fibres a three-level pumping model is used (see Fig. 2).

In this figure, laser level E1 is also a ground-state one. For simplification we can assume that a three-level data model is the special case of a four-level

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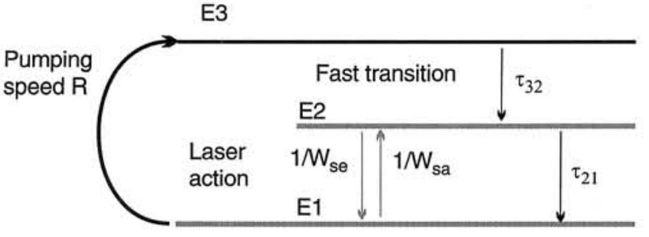

Fig. 2. Three-level optical pumping for Er-doped glass.

data model. Simplification can be written as $\tau_1 = \infty$ and $\tau_2 = \tau_{21}$. With additional assumption that τ_{32} is very small and all the atoms quickly change level E3 into level E2, level E3 can be omitted in the energy data model. It leads us into a two-level data model. Assuming further that probability of stimulated emission and signal absorption are equal, i.e., $1/W_{se} = 1/W_{sa}$ we get the simplified two-level data model shown in Fig. 3. An influence of this simplification will be discussed further.

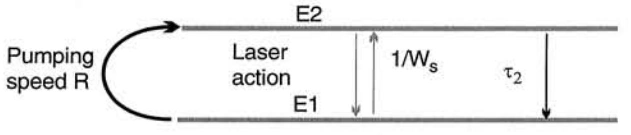

Fig. 3. A simplified two-level optic pumping.

For this situation the population of levels E2 and E1 can be calculated from Eq. (1) [5]

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_2} - N_2 W_s + N_1 W_s, \qquad N = N_1 + N_2 \tag{1}$$

where N is the total density of the dopant (concentration), N_1 is the dopant atoms density on E1 energy level, N_2 is the dopant atoms density on E2 energy level, W_s is the density of probability of the passage between levels E2 and E1 for the signal waves.

In the steady state the derivative from the level side of Eq. (1) equals 0. Introducing the dependencies according to [4]

$$R = N_1 W_p = N_1 \sigma_p \phi_p = N_1 \sigma_p \frac{P_p}{A_p h f_p},$$

$$W_s = \sigma_s \phi_s = \sigma_s \frac{P_s}{A_s h f_s},$$

where W_p is the density of probability of the passage between states E1 and E2 for the set length of the pump's wave; σ_s, σ_p are the active cross-sections for the passages from state E2 to E1 and from state E1 to E2, respectively; ϕ_s, ϕ_p are the radiation currents of the signal and the pump, respectively; P_s, P_p are the signal and pump strengths, respectively; A_s, A_p are the signal and pump wave surfaces, respectively; f_s, f_p are the signal and pump frequencies, respectively; and h is the Planck constant. Referring signal and pump power to the saturation power

$$P_s^0 = \frac{P_s}{P_{snas}}, \qquad P_p^0 = \frac{P_p}{P_{pnas}}$$

we get the expression for the N_2 steady state

$$N_2 = \frac{N(P_p^0 + P_s^0)}{1 + 2P_s^0 + P_p^0} \tag{2}$$

The saturation powers are as follows

$$P_{snas} = \frac{A_s h f_s}{\sigma_s \tau_2}, \qquad P_{snas} = \frac{A_p h f_p}{\sigma_p \tau_2} \tag{3}$$

On the basis of the population of levels N_1 and N_2 we can assess signal and pump power by introducing implicit functions. With the assumption that both waves propagate in the same directions and omitting the spontaneous emission we can write the following equations of propagation

$$\frac{dP_p}{dz} = -\sigma_p N_1 \Gamma_p P_p, \qquad \frac{dP_s}{dz} = s_s (N_2 - N_1) \Gamma_s P_s. \tag{4}$$

Inducing the absorption coefficients $\alpha_s = \alpha_s N \Gamma_s$ and $\alpha_p = \sigma_p N \Gamma_p$ for the defined length of the signal and pump wave, and taking into consideration early introduced dependencies, the equations can be written down in the convenient form [4]

$$\frac{dP_s}{dz} = \frac{(P_s^0 + 1)\alpha_p P_p}{1 + 2P_s^0 + P_p^0}, \qquad \frac{dP_s}{dz} = \frac{(P_p^0 + 1)\alpha_s P_s}{1 + 2P_s^0 + P_p^0}. \tag{5}$$

Equation (5) can be used for rough analysis of the working conditions and parameters of the optic amplifiers. They are convenient since the coefficients present in them can be easily identified experimentally. The active fibre producers often give those parameters in catalogues, e.g., Fibrescope Facts gives the following specification of the singlemode fibre DF1500Fs: absorption for the pump of 980 nm, 4.5 dB/m ±20%; absorption for the signal 1530 nm, 4.5 dB/m ±20%; saturation power of the signal 1530 nm, 13 dBm.

To solve Eqs. (5) for DF1500F's fibre pump radiation saturation power was missed. Using the technological parameters and relations defining the saturation power we can assess saturation power for pump radiation. For the mentioned fibre the pump saturation power was taken as equal to 17 dBm. Characteristic set of power radiation distribution in the fibre is shown in Fig. 4.

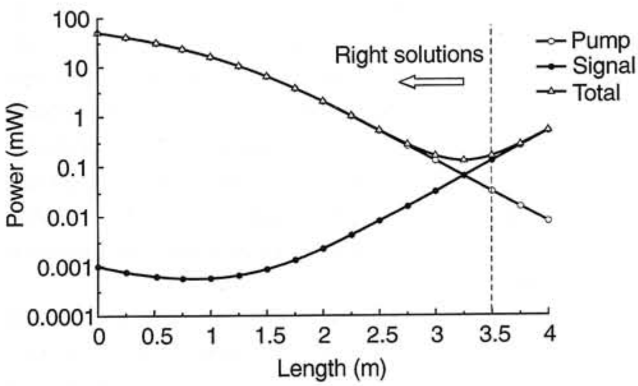

Fig. 4. Exemplary power distribution in a single mode fibre vs its length.

A domain of Eq. (5) defines monotonically decreasing range of the density N2. The relation of laser level population density, for the discussed case, as a function of fibre length is presented in Fig. 5.

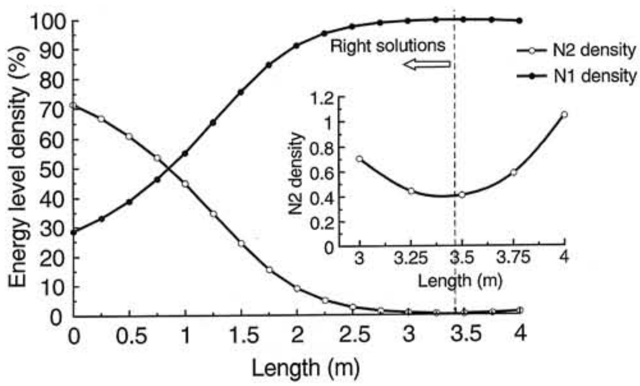

Fig. 5. Density of level population in singlemode optic fibre for the power distribution of Fig. 4.

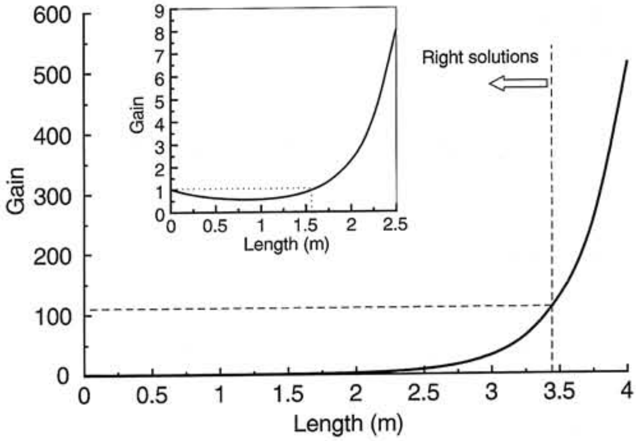

Fig. 6. Amplification of singlemode fibre vs its length.

It results from the data presented in Fig. 5 that solution of the propagation equations is proper for the fibre of length of 0–3.42 m. Self-increase in the level distribution density of level E2, outside the solution, results from the simplifications assumed for the theoretical model, and especially from the assumption of the same values of active cross-sections for emission and absorption effect. To have the amplification characteristics as a fibre length function it is enough to relate the power of the signal vs length to its initial value. For the analysed example the relation of amplification is shown in Fig. 6.

It results from this relation that the fibre length for amplification is also limited from lower limit by the range in which the signal is attenuated in spite of pumping. Limitation from upper limit of the fibre length does not result directly from the simplified model. In reality the fact of differences in active cross-section coefficients and signal attenuation in the background losses (typical for ordinary fibres) makes that amplification characteristics as a function of fibre length has its maximum. In the discussed data model the maximum amplification occurs at the end of the domain of Eq. (5).

3. Comparative analysis of singlemode and multimode amplifiers

Theoretical differences in the working conditions of singlemode and multimode amplifiers described on the basis of the previous chapter description are limited to other classification of the absorption coefficients and mode area of signal and pump wave. For multimode amplifiers the absorption coefficients are as follows: $\alpha_s = \sigma_s N$, for the signal wave; $\alpha_p = \sigma_p N$, for the pump wave. In both cases $\Gamma = 1$.

In general case the coefficients can be defined from the relation [6]

$$\alpha_s = \sigma_s \int_0^{2\pi} \int_0^{\infty} i_k(r,\theta) n(r,\theta,z) r dr d\theta,$$

for the signal wave, and

$$\alpha_p = \sigma_p \int_0^{2\pi} \int_0^{\infty} i_k(r,\theta) n(r,\theta,z) r dr d\theta,$$

for the pump wave, where i_k is the k-mode power distribution, n is the volume distribution of dopant function.

For singlemode amplifiers with homogenous doping the integral coefficient $\Gamma(\lambda)$ is the indicator of power distribution of the mode and the density of the dopant distribution (λ is the length of the wave). In this case the absorption coefficients are as follows: $\sigma_s = \sigma_s N\Gamma_s$ for the signal wave; $\alpha_p = \sigma_p N\Gamma_p$, for the pump wave. The coefficient Γ can change depending on the workmanship of the active optic fibre, the typical range of its values equals 0.2-0.6 [3]. The surface of pump and wave signals for multimode amplifiers can be assumed as equal to the surface of the core πb^2, where b is the core radius. A surface of singlemode amplifier of the base mode can be determined using the equation [4]

$$i(r) = \frac{1}{\pi}\left[\frac{v}{bV}\frac{J_0\left(\frac{ur}{b}\right)}{J_1(u)}\right]^2, \text{ for } r < b \qquad (7)$$

where

J_0, J_1 – Bessel's functions,

$V = \dfrac{2\pi b}{\lambda_k \sqrt{(n_{core}^2 - n_{clad}^2)}}$,

$v = 1.1428\ V - 0.9960$, and

$u = (V^2 - v^2)^{1/2}$.

For the analysed singlemode fibre the surface of the pump mode λ_p = 980 nm was assessed as $A_p = \lambda(3.64\ \mu m)^2$, and for the signal λ_s = 1550 nm as $A_s = \pi(6.40\ \mu m)^2$. The result of assumed diameter of singlemode fibre as 8 μm is $\Gamma_p = 0.21$ and $\Gamma_s = 0.64$. To compare quantitative singlemode and multimode amplifier parameters, the theoretical multimode fibre was constructed with the same amount of dopant as for the analysed example of singlemode fibre. To determine parameters of multimode fibre, the proportions for absorption and power assessment were used. The core diameter was assumed as equal to 50 μm. On this background we can give the following specification of the sample active multimode fibre: absorption coefficient for the pump 13.42 m^{-1} (singlemode 2.818 m^{-1}, Ref. 8); absorption coefficient for the signal 4.40 m^{-1} (singlemode 2.818 m^{-1}, Ref. 8); pump saturation power 9456 mW (singlemode 50.119 mW); and signal saturation power 1218 mW (singlemode 19.953 mW).

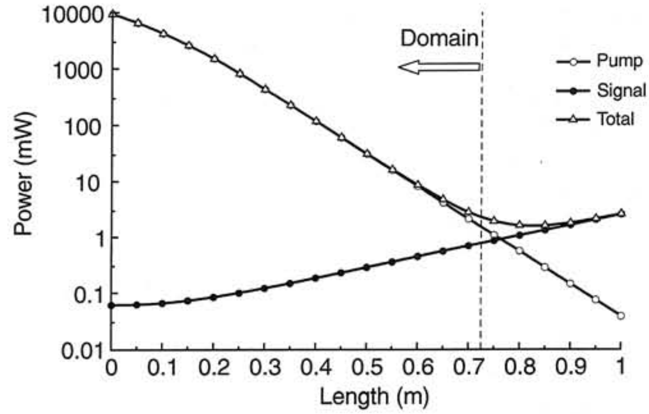

Fig. 7. Exemplary power distribution in a multimode fibre vs its length.

To ensure similar density of the pump and signal in singlemode and multimode fibre, the assumed pump power was 9434 mW and signal power 0.061 mW. For the above taken parameters of multimode fibre Eq. (5) was solved. The results are shown in Figs. 7, 8, and 9.

The relations of power propagated in the fibres are of similar character. They differ significantly in

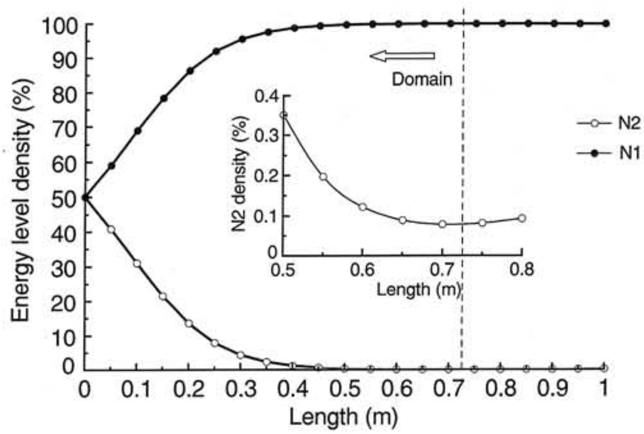

Fig. 8. Distribution density of energy level in a multimode fibre for the power distribution of Fig. 7.

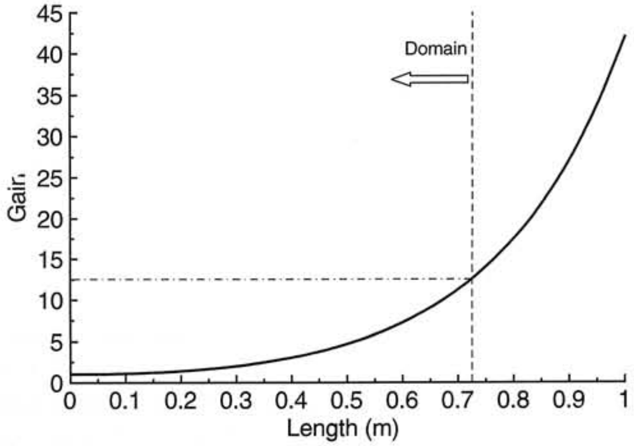

Fig. 9. Amplification in multimode fibre vs length.

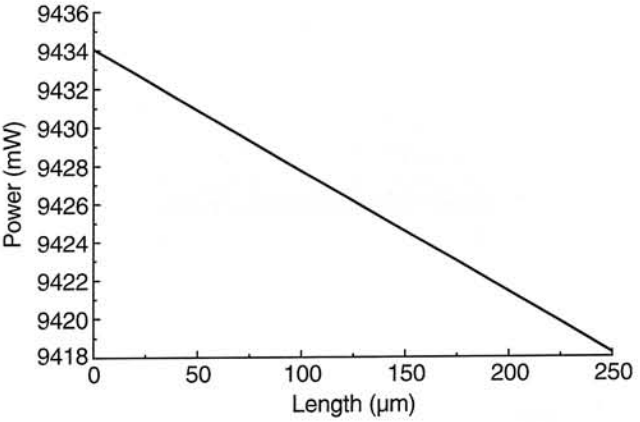

Fig. 11. Power distribution of initial 250 μm of multimode fibre.

lengths of the fibre at which the amplification takes place. The multimode fibre amplifies at the length of 0.72 m and singlemode fibre at 3.42 m. The density distribution of the energy levels indicates that the inversion level in a multimode fibre is significantly lower than in a singlemode one.

For the multimode fibre the amplification characteristics, presented in Fig. 10, does not show its minimum as its length function. In spite of retaining, the same level of doping and power distribution, the maximum multimode amplification is significantly lower and for the described case equals about 12.5, for singlemode fibre it equals about 103.

4. Discussion

The power characteristics presented in Figs. 4 and 7 indicate that in the fibres we can see the power losses. In the convention of the accepted two-level data model (Fig. 3) we can interpret them as radiative transitions. In reality there occur radiative, non-radiative, and intermediate-level transitions. Radiative transitions mean radiation of the photon from the core area in any direction [5] as it was shown in Fig. 10.

The case when the angle β_2 of the radiated photon, in relation to the optic axis of the fibre, is lower than critical angle of propagation means, that this photon joins the signal as a noise. When the angle β_2 is bigger than critical angle the photon leaves the core of the fibre. Then, it can be absorbed in the coating or the fibre screen or can leave the fibre.

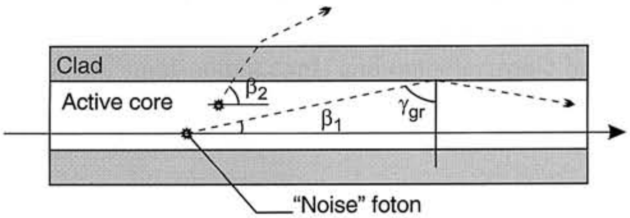

Fig. 10. Recombination radiation of photons.

The non-radiative and intermediate-level transitions mean heating of the fibre core. It is a very disadvantageous effect because of temperature increase in the fibre. Such effect is especially observed is the initial part of a fibre.

For both cases the analysis of total power distributions in the head of fibre is very important. The total power distribution propagated by the head of the analysed multimode fibre is presented in Fig. 11.

The loss in the initial fibre section of 250 μm was 15.84 mW. Referring to the radiative transitions [9], and assuming further that the propagation angle equals 17° it may mean generation of 15 mW×17/360 of the noise transmitting itself according to the signal in the initial part of the fibre. The noise level is 10 times higher than the input signal.

Another problem is effect of non-radiative and intermediate-level transitions that cause heating of a fibre core. Assuming that they amount 1% of the power losses we can assess a temperature distribution. To achieve it, the method of finite elements was applied. Taking axial symmetry of a fibre, the temperature distribution was assessed according to the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho r\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial \zeta}\left(p\frac{\partial T}{\partial z}\right)=-q, \tag{8}$$

where r, z are shown in Fig. 12, T is the temperature, ρ is the fibre heat conductivity, and q is the heat emitted in the area.

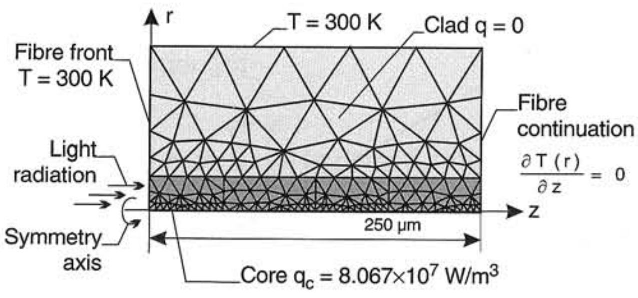

Fig. 12. Boundary conditions of the solution.

Distribution of power loss in the initial part the fibre (Fig. 11) can be approximated with a straight line with mean-square error equals 1.6×10^{-3}. On this basis we can assume, with good approximation, that the power loss in the initial part of the fibre does not depend on its length and equals 0.1584 mW. For further calculations was assumed: the clad diameter of 250 µm, heat conductivity of the fibre (as for glass) $\rho = 0.00922$ WcmK, and the following boundary conditions: constant temperature at the fibre surface equal to 300 K, no heat flux in the ending part of the analysed zone. The boundary conditions of the solution and the discretisation net for finite elements are presented in Fig. 12. The obtained results are presented in Figs. 13 and 14.

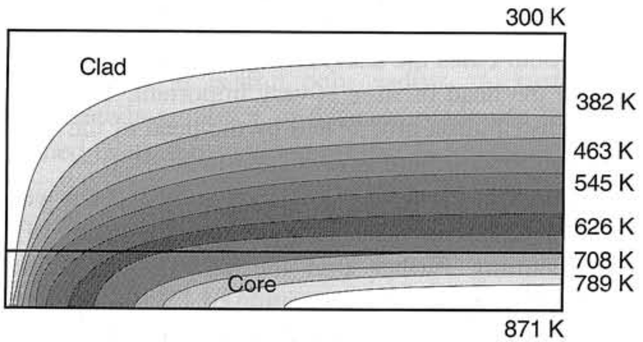

Fig. 13. Temperature distribution in the initial part of multimode fibre power pumping of 9434 mW.

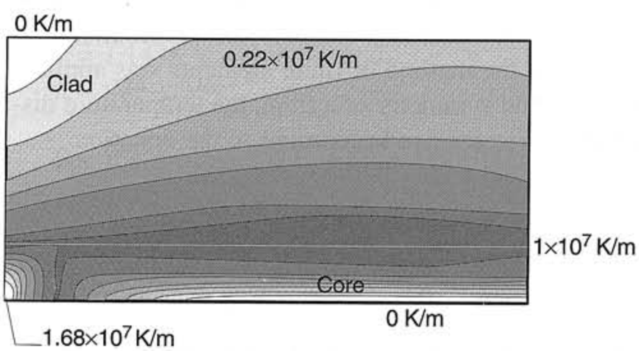

Fig. 14. Gradient of temperature distribution in the initial part of multimode fibre.

It results from the calculations that temperature of the core of multimode fibre pumped with 10 W power can reach 871 K.

5. Conclusions

For analysis of multimode active fibres we can use analogies present in the theoretical data model for singlemode fibres. However, it should be noticed that in multimode fibres thermal and noise phenomena could occur that they do not occur in such intensity in similarly doped singlemode fibres. Such phenomena can significantly reduce the allowed working points for multimode amplifiers. Thus, analysis of amplifiers based on active multimode fibres should be carried out before their design and manufacturing.

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