Contributed paper

OPTO-ELECTRONICS REVIEW **8**(2), 147–151 (2000)

Radiation amplification identification in optical fibre amplifiers

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The article presents theoretical background of radiation amplification in optic singlemode and multimode fibres and involves multimode fibre amplifier model worked out by the author. The analysis of amplification is based on the equations describing population of energetic levels and on the equations of radiation propagation. The possibilities of amplification identification of single and multimode amplifiers were checked by the authors. On these bases they evaluate the multimode amplifier pumped contradirectionally model. The method applied for this model could be easily implemented in singlemode amplifier pumped contradirectionally parameters calculation.

Keywords: optical fibre amplifiers, singlemode fibres, multimode fibres, codirectional and contradictional pumping.

1. Introduction

Nowadays, optical fibre amplifiers are widely used. A basic amplifier parameter is gain. Many theoretical problems can be solved for the amplifier design. The first one is calculation of accurate gain that requires many parameters and effects in a system of many differential equations. There are two possibilities to solve this problem. First, is application of fast computer and second, application of amplifier model as simple as possible and perceive. For the second way, the questions of minimum number of needed parameters and possibility of their achievement arise. A lot of publications present single mode erbium doped fibre amplifiers pumped codirectionally [1,2] and for this configuration the gain identification is known [3]. Very little information is available about multimode amplifiers [4] and amplifiers pumped in other way, e.g., contradirectionally.

2. Theoretical background of amplification in active environment

One of the most important kinds of optical amplifiers are optical fibre amplifiers. The amplifying medium is rare earth doped (e.g., Er, Nd, Pr) and pumped fibre. The optical pumping depending on the used dopant can be described with the four–level or three–level energetic model. Both of these models can be transformed into two–level model with some simplifying assumption [5]. Such a model is presented in Fig. 1.

For this case population of the levels E2 and E1 can be determined by eguation

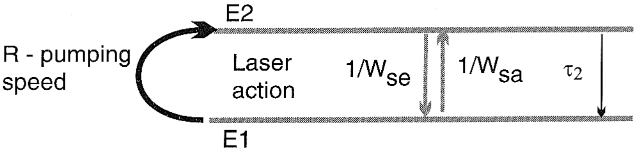

Fig. 1. Two-level model of optical amplification.

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_2} - N_2 W_{se} + N_1 W_{sa}, \qquad (1)$$
$$N = N_1 + N_2,$$

where N is the total density of the dopant (concentration), N_1 is the dopant atom density on the energy level E1, N_2 is the dopant atom density on the energy level E2, R is the speed of pumping, W_{sa} is the probability density of transition from E1 to E2 for the signal radiation, W_{se} is probability density of transition from E2 to E1 for the signal radiation.

In the steady state, the derivative on the left side of the first of Eqs. (1) equals zero. Introducing the relation (after Ref. 5)

$$R = N_1 W_{pa} = N_1 \sigma_{pa} \frac{P_p}{A_p h f_p},$$
$$W_{se} = \sigma_{se}\phi_s = \sigma_{se} \frac{P_s}{A_s h f_s}, \qquad (2)$$
$$W_{sa} = \sigma_{sa}\phi_s = \sigma_{sa} \frac{P_s}{A_s h f_s},$$

where W_{pa} is the probability density of transition from E2 to E1 for the pump wavelength, σ_{pa} is the transition cross–section from E1 to E2 for the pump wavelength, σ_{sa} is the transition cross-section from E1 to E2 for the signal wave-

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length, σ_{se} is the transition cross-section from E2 to E1 for the signal wavelength; ϕ_s, ϕ_p is the radiation flux of signal and pump, respectively; P_s, P_p is the signal and pump power, respectively; A_s, A_p is the signal and pump wave surface, respectively; f_s, f_p is the signal and pump wave frequency, respectively; h is the Planck constant, the density of energy levels population as a function of power in the fibre can be assessed. To assess amplification, the calculations should include propagation equations of the pump and signal.

$$\frac{dP_p}{dz} = (-\sigma_{pa}N_1 - \alpha_{pt})\Gamma_p P_p, \qquad (3)$$
$$\frac{dP_s}{dz} = (\sigma_{se}N_2 - \sigma_{sa}N_1 - \alpha_{st})\Gamma_s P_s,$$

where z is the fibre axis coordinate, α_{pt} is the fibre core attenuation, regardless of active dopant for pump radiation, α_{st} is the fibre core attenuation, regardless of active dopant for signal radiation, Γ_p is the pump mode and dopant distributions overlapping integration factor, Γ_s is the signal mode and dopant distributions overlapping integration factor.

Precise solution of Eqs. (1) and (3), with regard of dependency (2), is possible with numerical methods [6]. The results of consideration are pump and signal power characteristics and level population density along the symmetry axis of active fibre. Advantage and disadvantage of the above mentioned solution is the fact that it is based on technological data. The advantage is observable when there is a need to project and produce the fibre for a particular application, the disadvantage is observable when the purchased fibre is to be used in non-standard applications. This happens since the fibre manufactures do not reveal all the parameters of the fibres they produce. In this case there is a problem of assessing the fibres' minimal number of parameters allowing identification of amplification and then selection of the appropriate points of working conditions. This identification is based on analysis of the characteristics of propagated power and energetic level population along the fibre's axis for the defined working point. For this reason, because of the propagated modes' number in the active fibre, the appropriate mathematical amplifiers models should be used.

3. Single-mode fibre

Identification of active single-mode amplification is made difficult by the fact that to increase pumping efficiency, producers try to make the concentration distribution of the active dopant to be in accordance with the power distribution of the pump propagating in the fibre. It makes the mathematical model of the amplifier more complicated. In addition, for the singlemode amplifier, the wave surface of the signal and the pump (A_s, A_p), are not the same as the intersection of the fibre's core. They have to be defined numerically. The task of identification is made much easy by the fact that the following parameters of the fibre can be measured: absorption coefficient for pump length as well as signal and value of saturation power for the same wavelength. Typical values are: saturation power for signal 15 mW, for pump 30 mW, the absorption coefficients for pump and signal 2.8 m^{-1}). These parameters allow for identification of amplification. Using them and the definition of saturation power according to Ref. 7,

$$P_{snas} = \frac{A_s hf_s}{\sigma_s \tau_2}, \qquad (4)$$
$$P_{pnas} = \frac{A_p hf_p}{\sigma_p \tau_2},$$

where P_{snas} is the saturation power of signal wave, P_{pnas} is the saturation power of pump wave, introducing the constants

$$P_s^o = \frac{P_s}{P_{snas}},$$
$$P_p^o = \frac{P_p}{P_{pnas}},$$

and assuming that, the probability density of passage from the state E1 to E2 and from E2 to E1, for signal radiation, are equal to each other and additionally the optic fibre without doping does not introduce attenuation singlemode amplifier model can be written down as the system of equations (after Ref. 7)

$$N = N_1 + N_2$$
$$N_2 = \frac{N(P_p^0 + P_s^0)}{1 + 2P_s^0 + P_p^0},$$
$$\frac{dP_s}{dz} = -\frac{(P_s^0 + 1)\alpha_p P_p}{1 + 2P_s^0 + P_p^0}, \qquad (5)$$
$$\frac{dP_s}{dz} = -\frac{(P_p^0 - 1)\alpha_s P_s}{1 + 2P_s^0 + P_p^0},$$

where α_p is the pump radiation absorption coefficient, α_s is the signal radiation absorption coefficient, which is enough to amplification identification and assessment of the work-

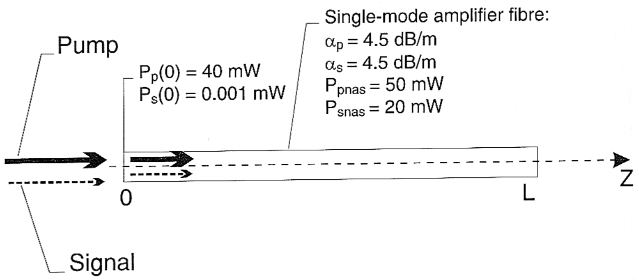

Fig. 2. Exemplary project's parameters of the singlemode amplifier.

ing point of single mode amplifier. Disadvantage of such a solution are significant simplifications making it necessary to produce a simultaneous analysis of power and energy level distribution [1] since the second one defines the domain of the solutions.

The example solution of Eq. (5), according to Ref. 4, for the amplifier with the parameters presented of picture 2, are shown in Figs. 3 and 4.

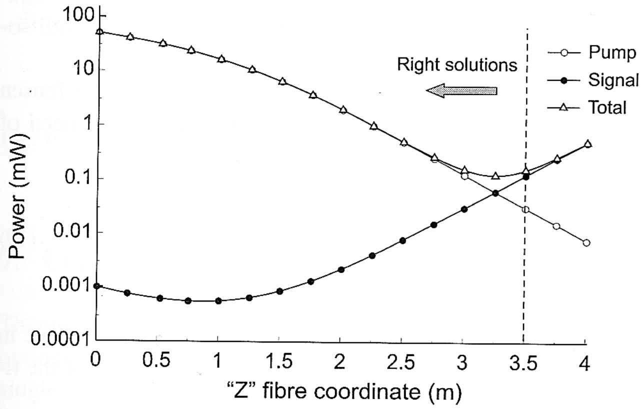

Fig. 3. Power distribution in a singlemode fibre based on Fig. 2.

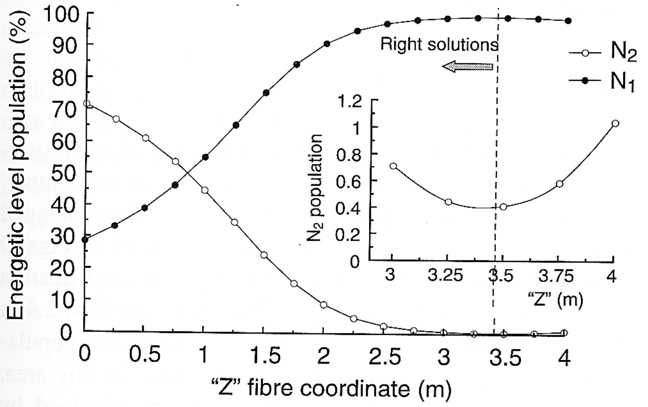

Fig. 4. Energy level population density in singlemode fibre for power distribution from Fig. 3.

4. Multimode fibre

For multimode fibres, with homogeneous dopant, the identification can be based on more precise mathematical model with equations (1), (2), and (3). In this case the solution simplifies the fact that the wave and pump signal wave surface (A_s, A_p) overlap with the core intersection and with themselves. So, Eq. (1) can be written as

$$N_2 = \frac{\sigma_{pa}\dfrac{P_p}{f_p} + \sigma_{sa}\dfrac{P_s}{f_s}}{\sigma_{pa}\dfrac{P_p}{f_p} + \dfrac{hA}{\tau_2} + \sigma_{se}\dfrac{P_s}{f_s} + \sigma_{sa}\dfrac{P_s}{f_s}}, \qquad (6)$$

where A is the fibre core surface.

Using Eq. (3), with overlapping integration factors equal to 1, we get the multimode amplifier model. For the analysis based on simplified multimode model, analogue to singlemode one, experimental assessment of saturation power could be difficult due to its high value. The proposed model does not cause the problem with assessment of experimental saturation power. So, to identify the amplification it is enough to assess experimentally a core diameter and attenuation for pump or signal radiation. After assessment of the core diameter, the identification can be done using the tabulated physical data of the active cross-sections for the pump and signal waves (σ_{sa} and σ_{pa}). Using them and the measured absorption coefficient and the dependency

$$\alpha_{sa} = \sigma_{sa}N, \quad \text{or} \quad \alpha_{pa} = \sigma_{pa}N \qquad (7)$$

we can assess a dopant concentration.

Because of technological level, the coefficients of fibre core attenuation α_{pt} and α_{st} (so–called background attenuation) can be assumed the same as for fibres with no dopant of the particular producer.

The next problem of amplifiers identification is their configuration. The optical amplifiers can be pumped codirectionally and contradirectionally.

5. Multimode fibre pumped codirectionally

An exemplary identification of amplification in multimode fibre pumped codirectionally, based on Eqs. (3), (6), and (7) is presented in Figs. 5, 6, and 7. The standard procedures can be applied for solution of differential equation.

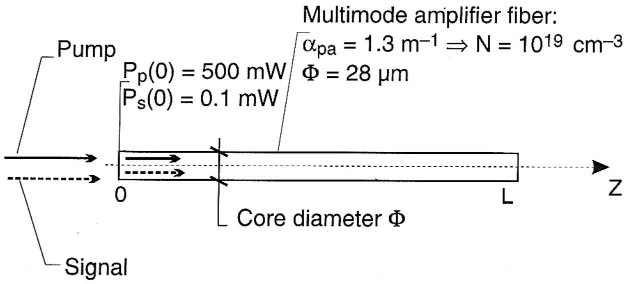

Fig. 5. Example of codirectionally pumped multimode amplifier.

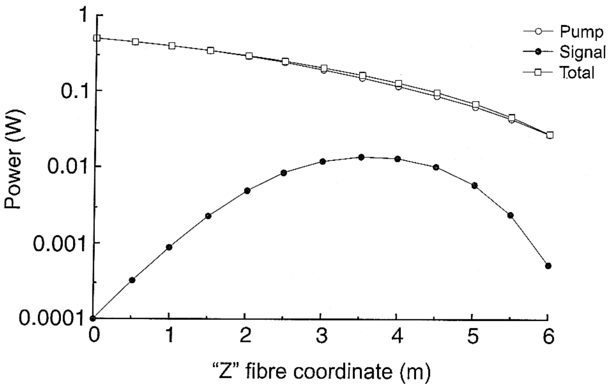

Fig. 6. Power distribution of multimode fibre based on Fig. 5.

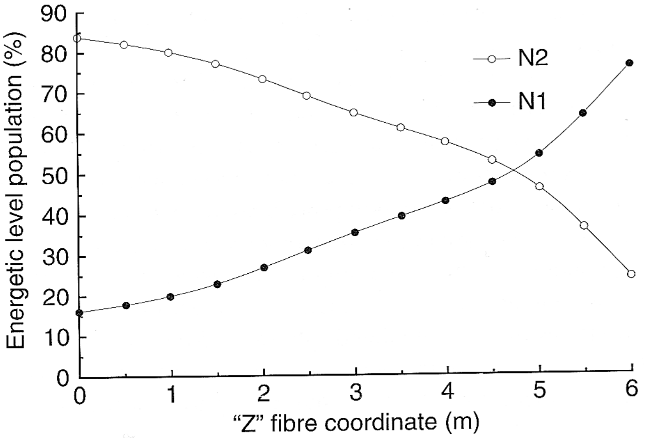

Fig. 7. Population density of energetic level in multimode fibre for the power distribution from Fig. 6.

Maximal amplification in this case equals ~120 and it is valid for the fibre of 3.6 m length. In the amplification area, over 60% of atoms are on the level E2.

6. Multimode fibre pumped contradirectionally

The example of amplifier with previously used multimode fibre pumped contradirectionally is described in Fig. 8. In this case to assess the amplifier's characteristics the left side of the first of Eq. (3) has to be multiplied by –1. It leads to Eq. (8). Full boundary conditions at the fibre beginning, necessary to use the standard solution procedures of the differential equations are not known.

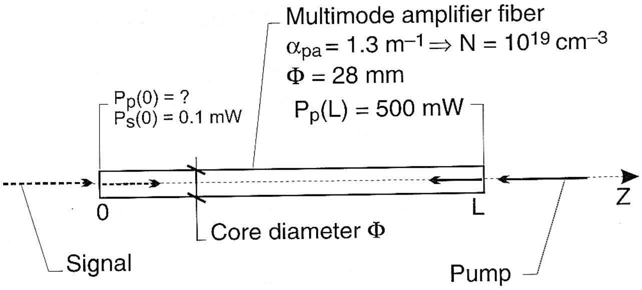

Fig. 8. Example of contradirectionally pumped multimode amplifier.

$$\begin{aligned}
&\frac{dP_p}{dz} = -1*(-\sigma_{pa}N_1 - \alpha_{pt})P_p\,,\\
&\frac{dP_s}{dz} = (\sigma_{se}N_2 - \sigma_{sa}N_1 - \alpha_{st})P_s\,,\\
&P_p(z=0) = ?\\
&P_s(z=0) = 0.1\ mW,\\
&P_p(z=L) = 500\ mW.
\end{aligned} \tag{8}$$

To solve the system of Eqs. (8) we can use iteration methods for solving the non-linear equation

$$P_{pe}(P_{pb}) - P_p(z=L) = 0, \tag{9}$$

where $P_{pe} \in (0; P_p(z=L))$ is the domain, P_{pe} is the pump's power at the fibre output, in the pump power function at the beginning of the fibre, assessed from Eq. (8), P_{pb} is the searched pump power at the fibre input.

To solve Eq. (9) the bisection or tangents methods can be applied. They allow for quick solving of differential Eq. (8) only at the ends of its domain ($z \in (0; L)$). This solution defines the boundary conditions necessary for full solution of Eq. (8) with standard methods.

To solve Eq. (9) we can also use the quick Steffensen method, although in this case Eq. (9) leads to the need of the full solution of Eq. (8) in every iteration

$$P_{pb\_i+1} = P_{pb\_i} = \frac{P_{pe}(P_{pb\_i})^2}{P_{pe}\left(P_{pb\_i} + P_{pe}(P_{pb\_i})\right) - P_{pe}(P_{pb\_i})}, \tag{10}$$

where P_{pb_i+1} is the pump power at the input of the fibre in iteration i + 1, P_{pb_i} is the pump power at the input of the fibre in iteration i.

The first approximate solution of Eq. (10) can be assessed, assuming that there is no signal in the fibre, with the equation

$$P_{pb\_0} = P_p(z=L)\,e^{-\alpha_p L}\,.$$

The fact that both ways of solving of Eq. (8) take similar time for the calculations, it seems to be interesting. The amplification characteristics of the fibre pumped contradirectionally (from Fig. 8) are presented in Figs. 9 and 10. In this configuration the maximum amplification equals about 8. It can be seen in Fig. 9 that the first 2.4 m of the fibre brings loses. It is caused by too weak population of the laser level E2, i.e., below 60% in this area. Maximal amplification in the fibre can be obtained by shortening of its length. Leaving the first 2.4 m of the fibre, the amplification shall be about 120, i.e., the same as in the case of codirectional pumping.

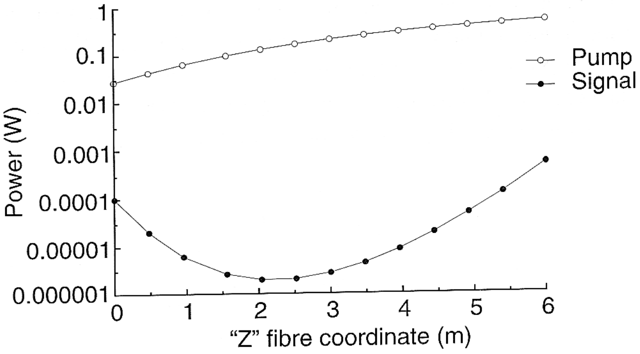

Fig. 9. Power distribution in a multimode fibre taken from Fig. 8.

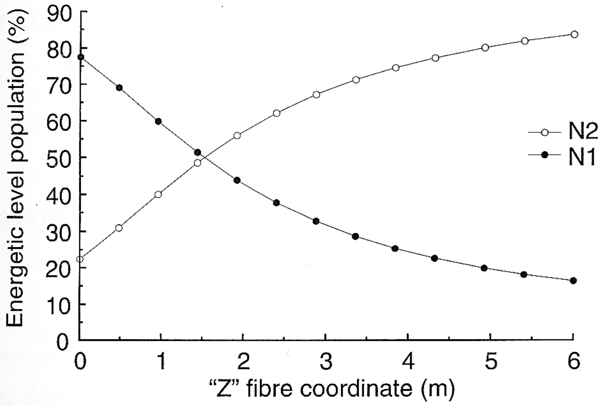

Fig. 10. Population density of energetic level in a multimode fibre taken from Fig. 8.

7. Conclusions

Theoretical backgrounds of the radiation amplifier in the optical fibres enable identification of amplification in singlemode and multimode amplifiers. Identification in both cases requires simultaneous measurement of fibre's parameters and calculations according to the presented models. For a singlemode fibre, four parameters have to be measured: saturation power, and attenuation coefficients for the waves of length of the pump and signal. For multimode fibre, only two parameters have to be measured: core diameter and attenuation coefficient. Those measurements are elementary ones. The presented methods of identification allow for design of amplifiers with real active fibres, the introductory selection of their working points in two pumping configurations. It reduces expensive laboratory works.

It is interesting that maximum amplification in multimode fibre, pumped codirectionally and contradirectionally, is the same and happens for the same fibre length. The disadvantage of contradirectionally fibre pumping is the fact that the pump power leaving the fibre, in the direction of the signal source is relatively high and in the described case it equals about 100 mW. It forces the optical filter for pump wave implementation at the fibre input and this cause potentially input signal losses. Similar losses are also in the real codirectionally pumping due to signal and pump coupling into an active fibre. So, configuration of amplifier pumping depends on other parameters of optical device.

References

1. Bjarklev, *Optical Fibre Amplifiers: Design and System Applications*, Artech House, Boston, London, 1993.
2. Desurvir, *Erbium Doped Fiber Amplifiers*, John Wiley & Sons, INC, New York, 1994.
3. A. Saleh, "Accurate modelling of gain in erbium–doped fiber amplifiers", *IEEE Photon. Techn. Letters* **2**, 714–717 (1990).
4. M. Borecki and J. Kruszewski, "Analysis of working conditions of the optic amplifiers with multimode fibres", *Opto-Elec. Rev.* **7**, 197–202 (1999).
5. B.E.A Saleh and M.C. Teich, *Fundamentals of Photonics*, John Wiley & Sons, INC, New York, 1991.
6. R. Giles, "Modelling erbium–doped fibre amplifiers", *J. Lightwave Techn.* **9**, 271–283 (1991).
7. G.P. Agrawal, *Nonlinear Fiber Optics*, Academic Press, San Diego, 1995.